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I. - LES MODELES UNIFIES DES INTERACTIONS FAIBLES ET ELECTROMAGNETIQUE

Les interactions rencontrées en physique sont habituellement classées en quatre grandes catégories : interactions fortes, électromagnétique, faibles et gravitationnelle. Seules les trois premières sont actuellement accessibles à l'expérience en physique des particules.

Il existe maintenant des théories unifiées des interactions faibles et électromagnétique, renormalisables : ce sont les théories de jauge brisées spontanément ; le premier exemple en est le modèle de Weinberg-Salam [1].

La théorie V-A des interactions faibles peut se mettre sous une forme rappelant l'électrodynamique quantique : un boson vectoriel chargé, lourd, est couplé à un courant. Pour décrire dans un même formalisme les interactions faibles et électromagnétique, on est amené à imposer une invariance par rapport à un groupe de jauge non abélien, de type Yang et Mills. Tous les bosons de jauge sont alors sans masse, pas seulement le photon. L'invariance de jauge doit donc être brisée, pour se réduire à la seule invariance par rapport au groupe U (1) de l'électrodynamique. Une brisure explicite, par l'addition de termes de masse à la densité de lagrangien, ne préserve pas la renormalisabilité de la théorie.

Aussi utilise-t-on une brisure spontanée de symétrie. Pour des raisons techniques, on est conduit à introduire des champs scalaires, bien que rien n'indique leur existence. Les états d'énergie minimum sont dégénérés ; l'un d'eux, non invariant par la symétrie, est choisi comme étant "le vide". Les champs scalaires sont translatés de manière à ce que leur valeur moyenne dans le vide soit nulle. La symétrie de la densité de lagrangien subsiste, mais de manière cachée.

La brisure spontanée d'un groupe de symétrie interne dépendant d'un paramètre réel entraîne l'existence d'un scalaire de masse nulle, le boson de Goldstone [2]. Toutefois, dans le cas d'une symétrie de jauge locale, celui-ci est éliminé par le mécanisme de Higgs : le boson de jauge, de spin 1, acquiert alors un degré de liberté supplémentaire en devenant massif [3]. Le travail de 't Hooft a permis de montrer que de telles théories de jauge étaient renormalisables.

En plus du boson vectoriel chargé, une théorie renormalisable des interactions faibles et électromagnétique doit introduire d'autres particules, supposées lourdes : boson vectoriel neutre, ou nouveaux leptons (voir par exemple l'article de revue [4], qui présente une introduction générale aux théories de jauge). De nouveaux effets sont attendus, et ont été activement recherchés expérimentalement. La découverte des événements "courants neutres" induits par des neutrinos [5] a permis d'éliminer toute une classe de modèles [6, 7], en particulier celui de Georgi et Glashow, où le photon est le seul boson vectoriel neutre.

Le modèle de Weinberg-Salam, qui n'introduit pas de nouveau lepton, permet de rendre compte de nombreux résultats expérimentaux. Il est invariant par rapport au groupe de jauge SU(2)xU(1); les bosons vectoriels lourds $W_{\pm}$ et Z sont responsables respectivement des événements "courants chargés" et "courants neutres", tandis que le photon γ est couplé au courant électromagnétique.

Cependant, avant même les très récentes découvertes faites dans les anneaux de collision $e_+ e_-$, tous les problèmes n'étaient pas résolus. Sur le plan expérimental, une règle de sélection des interactions faibles n'est pas encore parfaitement comprise : il s'agit de la règle $|\Delta \vec{I}| = \frac{1}{2}$ non-leptonique. D'un point de vue théorique, plusieurs questions fondamentales restent posées, telles que : peut-on éviter de faire jouer aux polarisations gauche et droite des rôles aussi différents que dans le modèle de Weinberg ? Peut-on obtenir la brisure spontanée de symétrie de manière nécessaire, sans que les paramètres du modèle soient astreints à vérifier certaines inégalités ? Pourquoi le neutrino est-il de masse nulle, comme le photon ?

Nous allons maintenant présenter brièvement les principaux résultats obtenus ; pour plus de détails, on pourra se reporter aux articles inclus [4, 9, 10, 14, 15, 28, 29]. Nous insisterons en particulier sur le rôle que la supersymétrie peut être appelée à jouer dans la théorie des interactions faibles. Cette symétrie d'un type nouveau, introduite récemment par Wess et Zumino [8], associe dans une même représentation des bosons et des fermions ;

lorsqu'elle est brisée spontanément, il existe une particule de Goldstone, de masse nulle et de spin 1/2 [9]. En nous limitant aux leptons du secteur de l'électron, nous avons construit un modèle supersymétrique des interactions faibles et électromagnétique, où le neutrino et le photon forment une représentation de masse nulle de la supersymétrie [10] . Ils apparaissent ainsi comme deux états de spins différents d'une même particule !

II. - LA REGLE $|\Delta \vec{I}| = \frac{1}{2}$ NON-LEPTONIQUE.

Les théories unifiées doivent inclure les hadrons de manière à satisfaire aux règles de sélection des interactions faibles. Dans le modèle de Weinberg, on évite les courants neutres changeant l'étrangeté par le procédé de Glashow, Iliopoulos et Maiani [11], qui fait appel à un quatrième quark, dit "charmé". On doit aussi tripler le nombre de quarks, en introduisant un groupe SU (3)' "de couleur" [12].

Comme dans le modèle courant-courant de Cabibbo, il se pose le problème de la règle $|\Delta \vec{I}| = \frac{1}{2}$ dans les transitions faibles non-leptoniques changeant l'étrangeté : les amplitudes $|\Delta \vec{I}| = \frac{1}{2}$ et $|\Delta \vec{I}| = \frac{3}{2}$ sont toutes deux proportionnelles au sinus de l'angle de Cabibbo ; il serait nécessaire de renforcer la première, pour rendre compte des désintégrations non-leptoniques des mésons K. On peut faire appel à un mécanisme de renforcement dynamique, résultant par exemple d'une théorie de jauge non-abélienne des interactions fortes [13]. Mais l'idée d'une règle de sélection opérant au niveau fondamental est toujours à envisager.

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C'est ce point de vue que nous avions adopté dans l'article [14]. Antérieur à la confirmation de l'existence des événements "courants neutres", cet article est exposé dans le cadre du modèle de Georgi-Glashow [6], aujourd'hui abandonné. Ses conclusions restent toujours valables, si l'on choisit SU (2) x U (1) comme groupe de jauge, en modifiant le secteur leptonique : on peut, par exemple, associer à l'électron et au muon deux neutrinos lourds, comme dans l'article [15], ou encore deux leptons lourds chargés. Les quarks sont les neuf quarks à charge entière de Han-Nambu, correspondant à la symétrie SU (3) x SU (3)' [16]. Utilisant la notion de "couleur", mais non celle de "charme", nous pouvons obtenir une règle $|\Delta \vec{I}| = \frac{1}{2}$ fondamentale.

Les quarks de Han-Nambu, invariants par U (1), sont répartis en triplets et singulets de SU (2) comme indiqué dans l'article [14], de manière à satisfaire aux contraintes suivantes :

- charges entières fixées,
- la partie "non-colorée" (i.e. singulet de SU (3)') du courant faible chargé a la forme de Cabibbo,
- les couplages du Z conservent l'hypercharge, et ceux du $W_{\pm}$ sont tels que les transitions $K_L \to \mu \bar{\mu}$, $K_L \to \pi \nu \bar{\nu}$ sont interdites aussi à l'ordre $G\alpha$,
- la transition $K\bar{K}$ est interdite aux ordres G et $G\alpha$,
- soit $\mathcal{L}_{\Delta Y = \pm 1}$ la partie "non-colorée" de la densité

de lagrangien effectif responsable des transitions non-leptoniques $\Delta Y = \pm 1$. $\mathcal{L}_{\Delta Y = \pm 1}$ commute avec $F_k + F_k^5(t)$, où les F_k sont les générateurs d'isospin, et $F_k^5(t)$ les opérateurs axiaux associés. Ceci permet de conserver les relations de l'algèbre des courants, par exemple entre les désintégrations $K \to 2\pi$ et $K \to 3\pi$.

$\mathcal{L}_{\Delta Y = \pm 1}$, proportionnel à la partie "non-colorée" du produit des courants chargés, est la somme de deux termes ; le premier est donné par la théorie de Cabibbo, le second est proportionnel à la partie "non-colorée" du produit des courants chargés "colorés". D'où la possibilité de compenser l'amplitude $|\Delta\vec{I}| = \frac{3}{2}$, ou de renforcer l'amplitude $|\Delta\vec{I}| = \frac{1}{2}$, données par la théorie de Cabibbo. Dans le modèle proposé, l'amplitude $|\Delta\vec{I}| = \frac{3}{2}$ s'annule avec l'angle de Cabibbo θ_c (elle peut même être $\sim \frac{\theta_c^2}{2}$), tandis que l'amplitude $|\Delta\vec{I}| = \frac{1}{2}$ reste finie dans cette limite.

Le but du modèle est seulement de montrer que l'introduction de la couleur permet, sans recours au charme, de rendre compte des règles de sélection des interactions faibles. Il peut aussi suggérer l'existence d'un lien entre l'amplitude résiduelle $|\Delta\vec{I}| = \frac{3}{2}$ et la valeur non nulle de l'angle de Cabibbo.

III. - LA BRISURE SPONTANEE DE LA PARITE.

Les théories unifiées des interactions faibles et électromagnétique doivent rendre compte de l'existence simultanée d'un courant électromagnétique purement vectoriel, et de courants faibles chargés de la forme V-A. Dans le modèle de Weinberg, les parties gauche et droite du courant électromagnétique sont obtenues de manières totalement différentes : les champs spinoriels gauches appartiennent à des doublets de SU(2), les champs spinoriels droits à des singulets. Cette dissymétrie fondamentale est contraire à l'idée générale des brisures spontanées : à partir d'une densité de lagrangien ayant la symétrie la plus grande, il s'agit d'obtenir une théorie où seules sont conservées les symétries existant dans la nature.

Nous allons présenter un modèle où la parité est brisée spontanément, en même temps que l'invariance de jauge SU(2) x U(1). Décrit dans l'article [15] pour des neutrinos à quatre composantes, le modèle est repris dans la section 2 de l'article [10] pour des neutrinos à deux composantes : à chaque lepton chargé est associé son neutrino ν_L , et un neutrino lourd à quatre composantes ν'.*

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* Au lieu d'un neutrino lourd ν' , on pourrait introduire un lepton lourd chargé E_-. Les fermions sont alors répartis en un doublet et un singulet de spineurs de Dirac, de la manière suivante :

$$\begin{pmatrix} \nu \\ e \\ -L \end{pmatrix} + \begin{pmatrix} E \\ -R \end{pmatrix} \qquad \begin{matrix} E \\ -L \end{matrix} + \begin{matrix} e \\ -R \end{matrix}$$

Le secteur de l'électron est constitué par un spineur de Majorana λ , invariant par SU(2) x U(1), et un doublet de spineurs de Dirac Ψ ; sa composante chargée sera le champ de l'électron. On introduit aussi deux doublets de champs de Higgs, φ' et φ'', échangés par parité, se transformant comme Ψ par le groupe de jauge.

L'existence d'un neutrino de masse nulle amène à imposer à la théorie une invariance supplémentaire, par rapport au groupe des transformations Q :

$$\begin{cases} Q\,\Psi\,Q^{-1} = \Psi \\ Q\,\lambda\,Q^{-1} = e^{\gamma_5 \alpha}\lambda \end{cases} \qquad \begin{cases} Q\,\varphi'\,Q^{-1} = e^{-i\alpha}\varphi' \\ Q\,\varphi''\,Q^{-1} = e^{i\alpha}\,\varphi'' \end{cases} \tag{1}$$

On a utilisé une représentation de Majorana, où γ_5 est une matrice antisymétrique réelle, de carré - 1. Ainsi on restreint considérablement la forme générale de la densité de lagrangien, déjà invariante par les transformations de jauge et de parité ; on interdit en particulier le terme de masse du spineur de Majorana λ ; il sera commode de considérer le spineur de Dirac à deux composantes

$$\lambda_L = \frac{1 - i\,\gamma_5}{2}\,\lambda$$

Si les paramètres du potentiel des champs scalaires satisfont aux inégalités convenables, un et un seul de ces champs prend une valeur moyenne dans le vide non nulle ; nous choisissons par exemple φ'. L'invariance par parité, qui échange φ' et φ'', est brisée spontanément, en même temps que l'invariance de jauge.

Exprimons en fonction des champs physiques le doublet Ψ de spineurs de Dirac, et le singulet λ_L :

$$\Psi = \begin{pmatrix} \nu_L \text{ Sin } \beta + \nu'_L \text{ Cos}\beta + \nu'_R \\ e_- \end{pmatrix} \qquad (2)$$

$$\lambda_L = \nu_L \text{ Cos } \beta - \nu'_L \text{ Sin}\beta$$

On peut montrer que la masse du neutrino est identiquement nulle, tandis que celles de l'électron et du neutrino lourd vérifient la relation $m_e = m_{\nu'} \text{ Cos } \beta$, qui fixe $\beta \simeq \frac{\pi}{2}$ (Les expressions (2) ont aussi été obtenues avec un système différent de champs scalaires, mais sans brisure spontanée de la parité [17]).

On lit directement sur la formule (2) les caractères essentiels du modèle. Les courants faibles couplant chaque lepton chargé à son neutrino sont de la forme V-A. Le courant neutre faible électronique est purement vectoriel, comme le courant électromagnétique ; d'où l'égalité des sections efficaces de diffusion $\nu_\mu e$ et $\bar{\nu}_\mu e$:

$$\sigma_{\nu_\mu e} = \sigma_{\bar{\nu}_\mu e} = \sigma^{V-A}_{\bar{\nu}_e e} \, (2 \text{ Sin}^2 \theta - 1)^2 \qquad (3)$$

θ étant l'angle de mélange de SU(2) x U(1).

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Nous avons étendu le modèle aux hadrons en associant trois quarks charmés aux trois quarks p, n, λ. Nous les avons répartis en

doublets et singulets du groupe de jauge par analogie avec les leptons, mais d'autres solutions sont possibles, respectant aussi la symétrie entre V-A et V+A. Malgré cela le courant neutre faible n'est pas purement vectoriel (ceci est une différence importante avec les modèles du même type dernièrement construits [18], qui n'utilisent que des doublets de SU(2) : le courant neutre faible y est purement vectoriel, et les sections efficaces "courants neutres" neutrino-hadron et antineutrino-hadron sont égales).

Dans le modèle de la Réf. [15], les données expérimentales relatives aux collisions $\nu_\mu e$, $\bar{\nu}_\mu e$, $\bar{\nu}_e e$, ν_μ-hadron et $\bar{\nu}_\mu$-hadron sont compatibles, avec $\sin^2\theta$ de l'ordre de 0,3 à 0,4. Pour les quarks p,n ,λ ,p',n',λ', des trois couleurs, la somme des carrés des charges, qui intervient dans l'interprétation des expériences d'annihilation e_+e_- → hadrons, est égale à 4. Après la découverte des nouvelles résonances [19], le modèle a été repris par Barnett [20], qui a discuté ses implications dans ce domaine. Enfin, en physique atomique, aucune violation de la parité n'est attendue dans les expériences sensibles à la partie axiale d'un courant neutre faible électronique [21].

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L'invariance de jauge et la parité sont donc deux symétries pouvant être imposées à la densité de lagrangien , dans une théorie unifiée des interactions faibles et électromagnétique.

En relation avec la masse nulle du neutrino, nous avons introduit un groupe de transformations Q. Dans la partie suivante nous mettrons en évidence des analogies remarquables avec certaines théories supersymétriques [9, 22], où les transformations Q se définissent très simplement à l'aide de superchamps.

IV. - LA SUPERSYMETRIE.

Les progrès importants réalisés en physique des particules correspondent souvent à l'introduction d'une nouvelle symétrie, reliant des particules déjà connues et pouvant éventuellement en prévoir de nouvelles. Ainsi pour les hadrons, l'isospin fait intervenir un nucléon à deux états de charge, proton et neutron ; la symétrie SU(3), qui généralise la précédente, a conduit à la prédiction du Ω^-. Les théories de jauge des interactions faibles et électromagnétique amènent à associer neutrino et électron, ou encore photon et bosons intermédiaires.

Dans les cas précédents les multiplets sont constitués, ou bien de bosons, ou bien de fermions. Or, nous connaissons un cas où un boson et un fermion ont la même masse : il s'agit du photon et du neutrino. Si l'on peut considérer cette dégénérescence comme accidentelle, l'attitude opposée est elle aussi légitime.

Il est alors très utile de disposer d'une symétrie où les multiplets contiennent à la fois des bosons et des fermions ; c'est le cas de la symétrie "de superjauge", ou "supersymétrie", introduite par Wess et Zumino [8].

Ses générateurs, qui transforment les bosons en fermions et inversement, sont quatre opérateurs de type Fermi, désignés par Q_α .

Ils commutent avec les opérateurs de translation P_μ , et se transforment par le groupe de Lorentz, engendré par les $M_{\mu\nu}$, comme les composantes d'un spineur de Majorana. Ils satisfont aux relations d'anticommutation :

$$\left\{ Q_\alpha \ , \ \overline{Q}_\beta \right\} = - 2 \ (\gamma^\mu)_{\alpha\beta} \ P_\mu \qquad (4)$$

La structure associée aux quatorze opérateurs $M_{\mu\nu}$, Q_α et P_μ est celle d'une "algèbre de Lie graduée". Notons que les relations de commutation entraînent que toutes les particules d'un multiplet ont la même masse. La supersymétrie devra donc être brisée pour avoir un intérêt physique. Nous examinerons cette question dans la partie suivante.

Les transformations de supersymétrie agissent simplement sur les superchamps de Salam et Strathdee [23], définis sur un superespace : un superchamp est fonction de quatre coordonnées de type Bose, et de quatre coordonnées de type Fermi, appartenant à une algèbre de Grassmann ; ce sont respectivement les coordonnées d'espace-temps, et les composantes d'un spineur de Majorana.

Un superchamp peut être développé en série des coordonnées de type Fermi ; comme celles-ci anticommutent, la série ne contient qu'un nombre fini de termes. Leurs coefficients sont des champs de bosons et de fermions, appelés composantes du superchamp.

On peut construire des théories supersymétriques décrivant l'interaction de superchamps [9, 10, 22, 24 à 29]. Certaines des

composantes apparaissent alors comme des champs auxiliaires, pouvant s'exprimer en fonction des champs physiques à l'aide des équations du mouvement. La relation d'anticommutation (4) permet d'obtenir directement le contenu physique des représentations. Nous utiliserons celles qui sont connues sous le nom de multiplets "scalaires" ou "vectoriels".

Un multiplet scalaire décrit, comme champs physiques, un spineur de Majorana et deux scalaires réels ; un multiplet vectoriel de masse nulle, un vecteur et un spineur de Majorana ; un multiplet vectoriel massif, un vecteur, un spineur de Dirac et un scalaire réel.

L'invariance de jauge peut être définie de manière compatible avec la supersymétrie ; chaque champ de jauge W_μ est associé à un spineur de Majorana λ dans un multiplet vectoriel sans masse. Ainsi le modèle de Wess et Zumino [22] qui possède une invariance de jauge U(1), décrit l'interaction d'un multiplet vectoriel sans masse (W_μ, λ) avec un multiplet scalaire complexe $(\Psi, \varphi', \varphi'')$ où Ψ est un spineur de Dirac, φ' et φ'' deux champs scalaires complexes échangés par parité (un multiplet scalaire complexe est formé de deux multiplets scalaires réels).

Ce modèle est très semblable à celui que nous avions présenté dans la partie III. Plus précisément, un modèle analogue à ce dernier, ayant U(1) comme groupe de jauge au lieu de SU(2)xU(1), et invariant par parité et par les transformations Q, contient comme cas particulier le modèle supersymétrique [22] ; la supersymétrie

détermine complètement les couplages des champs scalaires entre eux, et avec les champs spinoriels, en fonction de la constante de couplage du groupe de jauge ; on pourra comparer les densités de lagrangien données par les formules (3), (17) et (18) dans la Réf. [10].

Le modèle supersymétrique est donc invariant par les transformations Q. Celles-ci sont toujours définies par la formule (1), mais peuvent maintenant s'exprimer très simplement à l'aide des superchamps.

V. - LA BRISURE SPONTANEE DE LA SUPERSYMETRIE.

Si nous voulons associer dans un même multiplet des particules de masses différentes, nous devons briser la supersymétrie. L'idée la plus séduisante est celle de brisure spontanée, mais celle-ci ne peut être obtenue aussi simplement que pour les symétries ordinaires.

Dans une théorie supersymétrique, la relation d'anticommutation (4) permet d'exprimer l'hamiltonien en fonction des générateurs Q_α [25] :

$$H = \frac{1}{4} \sum_{\alpha=1}^{4} Q_\alpha^{\,2} \qquad (5)$$

Si $|\Omega>$ est un état du vide supersymétrique, $Q_\alpha|\Omega>$ est nul, et $|\Omega>$ a une énergie nulle. Par contre, si $|\Omega'>$ est un état du vide non supersymétrique, on peut s'attendre à ce que son énergie soit positive (bien que la relation (5) doive alors être écrite sous une forme locale, qu'il n'est pas légitime d'intégrer).

S'il en est effectivement ainsi, un état du vide supersymétrique est toujours stable, ce qui semblerait interdire la brisure spontanée. Toutefois ce n'est pas le cas, car nous avons pu construire des modèles où il n'existe pas d'état du vide supersymétrique [9, 10, 28, 29].

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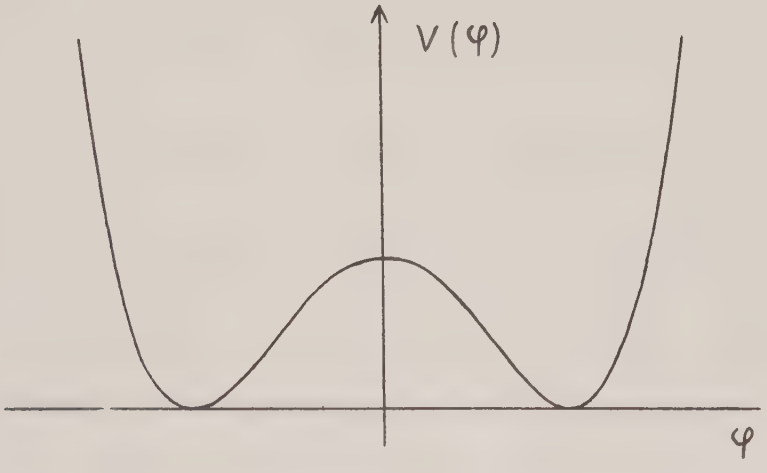
Il y a donc une grande différence entre la brisure spontanée de la supersymétrie et celle d'une symétrie de type usuel. Nous l'illustrerons sur le modèle de la Réf. [29], invariant à la fois sous les transformations de supersymétrie et les transformations de jauge du groupe local U(1). φ, complexe, est le seul champ scalaire, et le potentiel s'écrit :

$$V = \frac{1}{2} \left(\xi + e \varphi^{\dagger} \varphi \right)^2 \qquad (6)$$

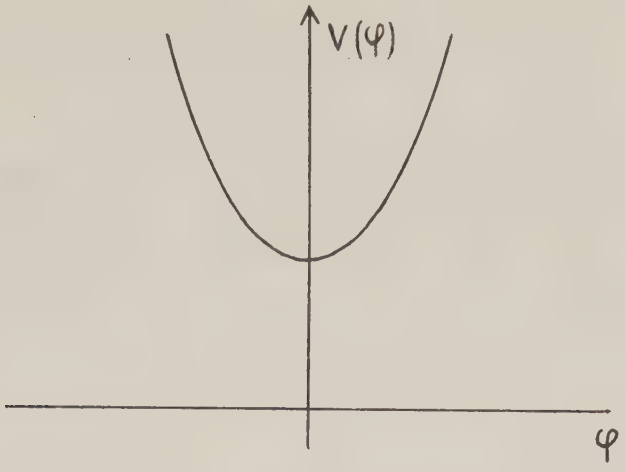
Si ξ e est négatif, il existe des états du vide supersymétriques, qui sont stables ; φ prend une valeur moyenne dans le vide non nulle et l'invariance de jauge est brisée spontanément (si φ ne prenait pas de valeur moyenne dans le vide, la supersymétrie serait brisée ; mais cet état est instable).



Cas où la supersymétrie est conservée.

Au contraire, si ξ e est positif, il n'existe pas d'état du vide supersymétrique ; φ ne prend pas de valeur moyenne dans le vide et l'invariance de jauge est conservée. La supersymétrie est

brisée spontanément, au sens où les équations du mouvement sont supersymétriques, mais le vide ne l'est pas.



Cas où la supersymétrie est brisée spontanément.

Remarquons que la relation entre forme du potentiel et brisure spontanée n'est pas la même pour la supersymétrie et pour une symétrie usuelle, comme l'invariance de jauge.

Lorsqu'une symétrie interne, engendrée par un opérateur scalaire de Lorentz, est brisée spontanément, il existe d'après le théorème de Goldstone un champ de masse nulle et de spin 0 [2]. De même lorsque la supersymétrie, engendrée par un opérateur de spin $\frac{1}{2}$, est brisée spontanément, il existe un champ de masse nulle et de spin $\frac{1}{2}$, qui a été appelé "spineur de Goldstone" [9, 25, 26]. Nous verrons plus loin si un tel champ est susceptible de représenter le neutrino.

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Nous montrons maintenant comment obtenir en pratique une brisure spontanée de supersymétrie, à l'approximation classique. Les modèles qui nous intéressent décrivent les interactions invariantes de jauge de multiplets vectoriels et scalaires.

Un multiplet vectoriel contient un champ auxiliaire D, un multiplet scalaire des champs auxiliaires F et G, qui s'expriment en fonction des champs physiques à l'aide des équations du mouvement ; le potentiel des champs scalaires s'écrit alors :

$$V = \frac{1}{2} \sum_k D_k^2 + \frac{1}{2} \sum_l (F_l^2 + G_l^2) \qquad (7)$$

On peut montrer que, pour que la supersymétrie soit brisée spontanément, il faut que l'un au moins de ces champs auxiliaires ait une valeur moyenne dans le vide non nulle, donc que le système :

$$D_k = F_l = G_l = 0 \qquad (8)$$

soit impossible (résoudre ce système, c'est trouver un état du vide supersymétrique, stable).

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Une première méthode, trouvée en collaboration avec Iliopoulos [9], s'applique aux théories possédant une invariance de jauge locale U(1). Le boson de jauge correspondant appartient à un multiplet vectoriel $\mathcal{V}$, qui contient un champ auxiliaire D. On ajoute à la densité de

lagrangien le terme ξD, qui brise la parité explicitement, mais de manière douce. Pour toute valeur non nulle de ξ , la supersymétrie est brisée spontanément, et il existe un spineur de Goldstone. Si de plus ξ est suffisamment grand, le boson de jauge devient massif, par brisure spontanée de l'invariance de jauge.

Notons aussi que dans la théorie supersymétrique le moment magnétique anormal du spineur chargé est identiquement nul [30]. C'est une raison supplémentaire de rechercher la brisure spontanée de la supersymétrie.

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Nous donnons à présent une deuxième méthode. Considérons les interactions renormalisables de n multiplets scalaires. Chacun d'eux contient deux champs scalaires physiques réels, que l'on peut récrire comme un champ complexe. Les n champs auxiliaires $F_1 + iG_1$ s'expriment comme des polynômes du second degré dans ces n champs physiques, dont on doit déterminer les valeurs moyennes dans le vide.

Le système (8), qui s'écrit

$$F_1 + iG_1 = 0 \qquad (9)$$

est un système du second degré, de n équations à n inconnues complexes. Il a en général une solution, déterminant un état du vide supersymétrique, stable : sauf dans des cas exceptionnels, il n'y a pas de brisure spontanée de supersymétrie.

Œpendant, en introduisant une nouvelle symétrie, on peut parfois contraindre les paramètres de la densité de lagrangien de manière telle que le système soit impossible. Il en est ainsi dans le modèle de la Réf. [28] ; globalement invariant par SU(2) x U(1), celui-ci décrit un triplet et un singulet de multiplets scalaires réels, et un doublet de multiplets scalaires complexes. Nous interdisons dans la densité de lagrangien les termes de masse directs, ainsi que la self-interaction du singulet ; nous utilisons pour cela une invariance par rapport à un groupe de transformations R, analogues aux transformations Q déjà mentionnées (de telles transformations nous permettront de définir un nombre leptonique conservé, dans le modèle semi-réaliste de la partie VII). La supersymétrie est alors brisée spontanément.

La détermination complète de l'état du vide nécessite des calculs d'ordre supérieur : à côté des bosons de Goldstone ordinaires associés à la brisure spontanée de la symétrie globale SU(2) x U(1), on trouve des bosons dont la masse, calculable, est nulle à l'ordre le plus bas.

L'étude faite s'étend immédiatement au cas où les invariances SU(2) ou U(1) sont réalisées localement. En particulier, nous pouvons obtenir une brisure spontanée de supersymétrie dans une théorie où le groupe de jauge est semi-simple.

VI. - INTERACTION SUPERSYMETRIQUE D'UN MULTIPLET VECTORIEL MASSIF.

Nous allons montrer comment donner une masse aux bosons vectoriels (i.e. briser l'invariance de jauge) de manière supersymétrique.

Rappelons qu'un multiplet scalaire décrit un spineur de Majorana et deux scalaires réels ; un multiplet vectoriel de masse nulle, un vecteur et un spineur de Majorana ; un multiplet vectoriel massif, un vecteur, un spineur de Dirac et un scalaire réel.

On s'intéresse initialement aux interactions invariantes de jauge de multiplets vectoriels sans masse, et de multiplets scalaires. Supposons qu'un boson vectoriel acquière une masse par brisure spontanée de la symétrie de jauge, un boson de Goldstone étant éliminé par le mécanisme de Higgs. Si la supersymétrie est conservée, tout un multiplet vectoriel devient massif, en s'unissant au multiplet scalaire engendré par le boson de Goldstone lors des transformations de supersymétrie ; les spineurs de Majorana de ces deux multiplets forment le spineur de Dirac du multiplet vectoriel massif.

Le modèle le plus simple décrit la self-interaction d'un multiplet vectoriel massif [29] . Il a déjà été mentionné dans la partie précédente ; nous étudions ici le cas où la supersymétrie est conservée, et l'invariance de jauge brisée spontanément.

Signalons que l'on peut aussi obtenir ce modèle, sans partir

d'une théorie de jauge, en recherchant pour un multiplet vectoriel massif une self-interaction supersymétrique sans terme quartique dans les champs spinoriels. On est conduit directement à la densité de lagrangien d'une théorie de jauge brisée spontanément, après élimination du boson de Goldstone. Elle est donnée par la formule (7) de la Réf. [29].

Le modèle décrit l'interaction d'un vecteur, d'un spineur de Dirac et d'un scalaire réel, de même masse.

Notons que :

- il existe, trivialement, un nombre fermionique conservé : nul pour le vecteur et le scalaire, il est égal à 1 pour le spineur de Dirac,

- l'énergie du vide est nulle ; un champ gravitationnel extérieur n'induit pas de terme cosmologique [31],

- il n'y a que deux paramètres : la masse, et la constante de couplage du groupe de jauge, e. Rappelons que le modèle de Higgs usuel [3] dépend aussi d'un paramètre de masse, de la constante de couplage e, mais encore d'un troisième paramètre arbitraire, coefficient du terme $(\varphi^\dagger\varphi)^2$ dans le potentiel. Dans le modèle supersymétrique, ce dernier paramètre est fixé, et égal à $\frac{e^2}{2}$.

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La renormalisabilité est détruite par l'anomalie due au couplage V-A du boson vectoriel. Mais en introduisant d'autres multiplets, on obtient facilement une compensation des anomalies. Nous pouvons par exemple ajouter un multiplet scalaire, pour retrouver le couplage purement vectoriel d'un spineur de Dirac ; le modèle est celui de la Réf. [9], où une R-invariance oblige la masse m à être nulle ; il ne dépend alors que des paramètres ξ et e, qui peuvent prendre toutes les valeurs non triviales ; dans tous les cas, le potentiel, à l'approximation classique, est positif, et conduit à une brisure spontanée de l'invariance de jauge : aucune inégalité n'est à imposer aux paramètres du modèle, ce qui est satisfaisant.

Dans la partie suivante, nous verrons un nouvel exemple de brisure spontanée de l'invariance de jauge, la supersymétrie étant conservée.

VII. - UNE THEORIE SUPERSYMETRIQUE DES INTERACTIONS FAIBLES ET ELECTROMAGNETIQUE DE L'ELECTRON ET SON NEUTRINO.

Nous connaissons à présent les mécanismes de brisure spontanée, pour la supersymétrie comme pour l'invariance de jauge. Nous allons les appliquer au cas du groupe de jauge SU(2) x U(1), et de manière à ce que le spectre de particules puisse recevoir une interprétation physique [10]. Nous procéderons en deux étapes, pour plus de clarté dans l'exposé.

a) Brisure spontanée de l'invariance de jauge, la supersymétrie étant conservée.

Soient $\vec{\mathcal{V}}$ et $\mathcal{V}'$ les multiplets vectoriels sans masse contenant respectivement les bosons de jauge de SU (2) et U(1) ; ils interagissent avec un doublet $\mathcal{D}$ de multiplets scalaires complexes, contenant en particulier les champs de Higgs qui prendront une valeur moyenne dans le vide non nulle ; g et g' sont les deux constantes de couplage. On introduit aussi un multiplet scalaire réel $\mathcal{S}$, invariant de jauge mais couplé à $\mathcal{D}$, avec la constante h. La densité de lagrangien comprend un terme proportionnel à la composante F de ce multiplet son coefficient s fixe l'échelle de masse.

Outre les invariances par rapport aux transformations de

jauge,de supersymétrie et de parité, on demande l'invariance par rapport à un groupe de transformations R ; celle-ci interdit, dans la densité de lagrangien, les termes de masse directs des multiplets scalaires $\mathcal{D}$ et $\mathcal{S}$, ainsi que la self-interaction de $\mathcal{S}$. On en déduira ultérieurement la conservation du nombre leptonique.

La supersymétrie, la parité et la R-invariance sont conservées, alors que l'invariance de jauge est réduite à un sous-groupe abélien U(1). Soit θ l'angle de mélange défini par $tg\ \theta = g'/g$ [1]. Les bosons vectoriels W_- et Z deviennent massifs, tandis que le photon γ reste de masse nulle.

Puisque la supersymétrie est conservée, trois multiplets vectoriels deviennent massifs en s'unissant à des multiplets scalaires. La théorie brisée décrit l'interaction des multiplets vectoriels massifs $\mathcal{V}_-$ et $\mathcal{V}_z$, du multiplet vectoriel de masse nulle $\mathcal{V}_\gamma$, avec un multiplet scalaire complexe. Ce dernier joue, pour une théorie supersymétrique, le rôle du scalaire de Higgs physique dans le modèle de Weinberg ; sa masse est proportionnelle à la constante de couplage h, celles de $\mathcal{V}_-$ et $\mathcal{V}_z$ étant proportionnelles à g et $\sqrt{g^2 + g'^2}$.

Le spectre de particules est le suivant :

$\mathcal{V}_z\ (Z\ ;\ E_o\ ;\ \chi)$

$(e_o\ ;\ \omega\ ,\ \varphi\ )$

$\mathcal{V}_-(W_-\ ;\ E_-,\ e_-\ ;\ w_-)$

$\mathcal{V}_\gamma\ (\ \gamma;\ \lambda_\gamma\ )$

W_- , Z et γ sont les bosons vectoriels.

η est un scalaire réel ; w_- , ω et φ des scalaires complexes.

E_-, e_- , E_o, e_o sont des spineurs de Dirac ; λ_γ est un spineur de Majorana (on pourra aussi bien considérer le spineur de Dirac à deux composantes $\lambda_{\gamma L} = \frac{1 - i\gamma_5}{2} \lambda_\gamma$).

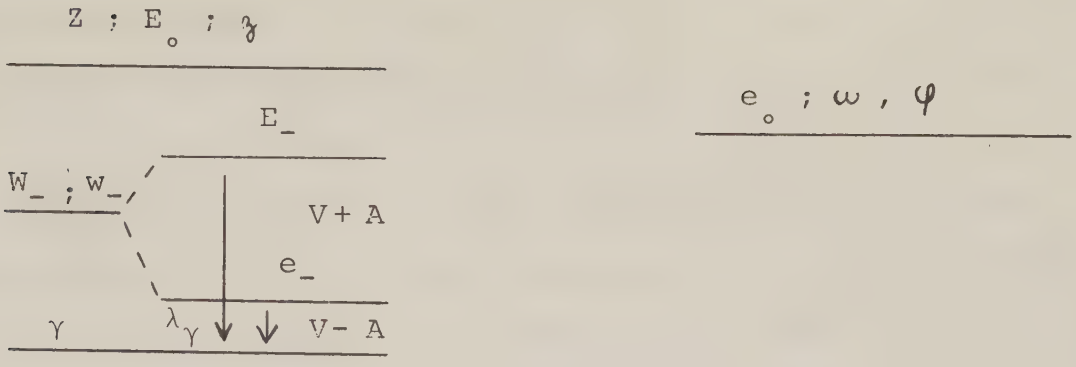
La R-invariance est encore une symétrie de la théorie brisée ; d'où la conservation d'un nombre quantique additif, porté par les spineurs (+1 pour e_- , e_o et $\lambda_{\gamma L}$; -1 pour E_- et E_o), mais aussi par le scalaire complexe ω (+ 2).

Supersymétrie et parité sont donc conservées, en même temps qu'un nombre quantique additif ; d'autres exemples sont facilement obtenus, par exemple en réduisant le groupe de jauge à SU(2), ou en se limitant à une invariance U(1).

b) Brisure de la supersymétrie.

Nous utilisons la méthode de la Réf. [9] : ajoutons à la densité de lagrangien le terme $\xi D'$, qui respecte les invariances par SU(2) x U(1), supersymétrie et R, et brise, explicitement mais de manière douce, la parité. Comme précédemment, l'invariance de jauge est réduite au sous-groupe U(1), et la R-invariance est conservée. Mais la supersymétrie est brisée spontanément : le spineur de Goldstone est le spineur de Majorana λ_γ , associé au photon dans un multiplet vectoriel.

Le spectre de masse devient, à l'approximation classique :



Les courants chargés couplant e_- et E_- à λ_γ sont respectivement de la forme V-A et V+A (rappelons que, lorsque ξ est nul, e_- et E_- sont dégénérés en masse, la parité est conservée et le courant chargé peut se récrire de manière purement vectorielle).

Nous interprétons e_- et E_- comme les champs de l'électron et d'un électron lourd ; $\nu_L = (\lambda_\gamma)_L$ est le champ de Dirac d'un neutrino à deux composantes, couplé à e_- et E_+ par des courants chargés V-A. γ désigne toujours le photon, couplé au courant électromagnétique, purement vectoriel ; toutes les autres particules ont des masses élevées.

Le moment magnétique anormal de l'électron, qui serait nul si la supersymétrie était conservée, est bien donné par $\frac{g-2}{2} = \frac{\alpha}{2\pi} + \ldots$, les corrections dues aux interactions faibles étant négligeables.

Les courants chargés couplant l'électron au neutrino sont de la forme V-A ; mais au boson vectoriel W_- est associé un boson scalaire w_- de même masse. Tous deux peuvent être échangés dans une

diffusion neutrino-électron, et l'on trouve que les sections efficaces de diffusion $\sigma_{\nu_e e}$ et $\sigma_{\bar{\nu}_e e}$ sont égales. L'existence de telles particules scalaires est une cause de difficultés lorsque l'on cherche à étendre le modèle de manière à inclure le secteur du muon.

Le nombre quantique conservé associé à la R-invariance est interprété comme étant le nombre leptonique. E_- est un lepton lourd de type Georgi-Glashow : e_- a nombre leptonique (+1), et E_- (-1). Il n'y a donc pas de vertex $\overset{e_-}{\underset{}{}}\!\!-\!\!\!\!\!\!\sim\!\!\!\!\!\!-\overset{E_-}{\underset{}{}}$ Z, mais il existe un vertex $\overset{e_-}{\underset{\omega}{}}\!\!-\!\!\!\!\!\!\sim\!\!\!\!\!\!-\overset{E_-}{\underset{}{}}$, le boson scalaire ω ayant un nombre leptonique 2.

Le modèle décrit aussi deux neutrinos lourds. L'un d'entre eux se mélange au neutrino, définissant un angle analogue au β de Georgi et Glashow [6] ; la supersymétrie lui impose d'être égal à l'angle de mélange θ du groupe SU(2) x U(1) [1].

Comment comprendre la petite masse de l'électron, dans un modèle incluant plusieurs leptons lourds ? Si l'on impose une invariance supplémentaire X, le paramètre δ est nul ; seuls subsistent ξ, qui fixe l'unique échelle de masse, et les trois constantes de couplage g, g', h. A l'approximation classique, l'invariance X est conservée et l'électron a une masse nulle ; la petite masse de l'électron apparaît donc liée à la brisure de cette invariance.

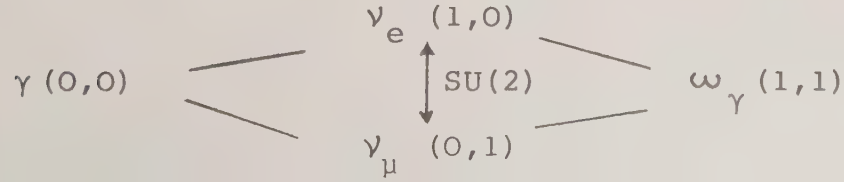
Dans le présent modèle, où le photon est associé au neutrino, spineur de Goldstone, en un multiplet vectoriel sans masse, nous nous sommes limités au secteur de l'électron. Il nous reste à examiner les problèmes qui se posent lorsque l'on cherche à aller plus loin.

c) Vers une théorie réaliste ?

L'idée que le neutrino est une particule de Goldstone a des conséquences expérimentales importantes, étudiées dans la Réf. [32]. L'invariance par supersymétrie entraîne l'existence d'un courant vecteur-spineur conservé ; lorsque la supersymétrie est brisée spontanément, ce courant contient un terme proportionnel au spineur de Goldstone. Par une méthode analogue à celle utilisée pour les pions de basse énergie, on étudie l'amplitude d'émission d'un neutrino dans un processus physique. Elle doit s'annuler en bout de spectre, dans une désintégration β , ce qui est contraire aux résultats expérimentaux. Il semble donc que le neutrino ne soit pas une particule de Goldstone.

Mais l'association du neutrino et du photon dans un même multiplet reste une idée séduisante. Toutefois, nous connaissons deux neutrinos, ν_e et ν_μ ; on pourrait penser à leur associer deux bosons vectoriels, dont une combinaison serait le photon, l'autre devenant massive. Mais indiquons une deuxième possibilité, qui nous semble préférable : aux deux neutrinos sont associés le photon γ et une particule scalaire complexe ω_γ. La symétrie électron-muon se traduit par une invariance par rapport à un groupe SU(2), dont $\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}_L$ est un isodoublet, tandis que γ et ω_γ sont des isosinglets.

Au groupe U(1) de l'électrodynamique correspond, non pas le seul photon, mais le multiplet de particules de charge nulle :



Les nombres quantiques électronique et muonique sont indiqués sur la figure : ω_γ a nombre électronique 1 et nombre muonique 1.

Il en résulte en particulier qu'un ω_γ ne peut être échangé entre des lignes d'électrons ou de muons, ou contribuer à la désintégration du μ . Les couplages de ω_γ font intervenir des particules lourdes, et la phénoménologie de basse énergie n'est pas modifiée (l'échange d'un ω_γ ne peut contribuer de manière importante au moment magnétique anormal de l'électron ou du muon, car on s'attend à ce que ses couplages avec les lignes de leptons chargés soient du type $1 \pm i\gamma_5$). Par contre, une paire de leptons lourds chargés pourrait être créée par l'échange d'un ω_γ dans une collision $e_+ e_-$.

La particule ω_γ est initialement sans masse. Contrairement au photon et aux neutrinos, elle est susceptible d'en acquérir une par brisure spontanée, mais peut-être n'est-ce qu'aux ordres supérieurs de la théorie des perturbations.

Ceci n'est qu'un schéma, à inclure dans un modèle réaliste. Mais il montre le type de conséquences que peut avoir l'application des idées de supersymétrie à la théorie des interactions faibles et électromagnétique.

VIII. - CONCLUSION.

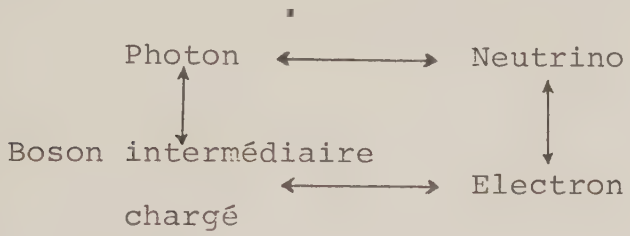
L'unification de la description des interactions faibles et électromagnétique dans une théorie renormalisable suppose l'existence de nouvelles particules, ayant des masses de l'ordre de cinquante GeV/c^2 ou plus, alors que le spectre n'est pas connu au-delà de quelques GeV/c^2.

Les résultats remarquables obtenus par les expérimentateurs, et les recherches en cours (propriétés des "courants neutres", recherche du "charme" et de "leptons lourds") sont essentiels pour déterminer une classe de modèles possibles. Mais l'accumulation des données expérimentales doit s'accompagner d'un effort théorique de structuration ; en particulier, de la recherche de la symétrie la plus grande.

L'application des idées d'invariance de jauge et de brisure spontanée aux interactions faibles et électromagnétique s'est déjà montrée extrêmement fructueuse. D'autre part, il est possible qu'aucune des deux orientations de l'espace, gauche et droite, ne soit initialement privilégiée ; nous l'avons illustré en construisant une théorie où l'invariance de jauge et la parité sont brisées spontanément.

La supersymétrie associe dans un même multiplet des bosons et des fermions : un superchamp, dépendant d'un point du superespace défini par ses coordonnées d'espace-temps et par de nouvelles coordonnées de type Fermi, décrit à la fois les deux types de particules.

Contrairement aux théories de jauge usuelles, qui dépendent de nombreux paramètres arbitraires, les théories supersymétriques sont très "rigides", et nous n'en connaissons pas actuellement qui soit réaliste ; en particulier, on semble encore loin d'arriver à décrire les quarks. Cependant nous avons pu construire un modèle, en nous limitant aux leptons du secteur de l'électron. Représentons, de manière très simplifiée, comment sont reliées les diverses particules par les transformations de jauge et de supersymétrie :



Dans une telle théorie "superunifiée", photon et neutrino, décrits par le même superchamp, apparaissent comme deux états de spin différents de la même particule.

Mais il existe deux neutrinos, et on pourrait être amené à leur associer, outre le photon, une particule scalaire de charge nulle de nombres électronique et muonique 1.

On ne peut encore dire si la supersymétrie est une invariance du monde physique, brisée spontanément ; mais elle fournit un cadre nouveau et élégant permettant de traiter de la même manière les bosons et les fermions.

L'algèbre des transformations de supersymétrie est une extension de l'algèbre des transformations de Poincaré. Elle a aussi une

interprétation géométrique, grâce au concept de superespace, et nous croyons que ce sont de telles symétries qui, comme l'invariance de jauge, jouent un rôle en physique. Enfin, le superespace peut se prêter à l'introduction de nouvelles structures, incluant la supersymétrie et la relativité générale.

Espérons pour finir que la conjonction des recherches théoriques et expérimentales permettra d'unifier la description de toutes (?) les interactions dans une théorie cohérente d'une grande beauté.

REFERENCES.

[1] S. Weinberg, Phys. Rev. Letters 19 (1967) 1264.
A. Salam, Proc. of the 8th Nobel Symposium (Wiley, NEW-YORK, 196

[2] J. Goldstone, Nuovo Cimento 19 (1961) 154.
J. Goldstone, A. Salam and S. Weinberg, Phys. Rev. 127(1962) 965

[3] F. Englert and R. Brout, Phys. Rev. Letters 13 (1964) 321.
P. W. Higgs, Phys. Rev. Letters 13 (1964) 508.
T. W. Kibble, Phys. Rev. 155 (1967) 1554.

[4] P. Fayet, Proc. of the 9th rencontre de Moriond, ed Tran Thanh V
(197

[5] P. Musset, Report to the 2nd Int. Conf. on elementary particles
at AIX-EN-PROVENCE, 1973.

[6] H. Georgi and S.L. Glashow, Phys. Rev. Letters 28 (1972) 1494.

[7] B. W. Lee, Phys. Rev. D6 (1972) 1188.
J. Prentki and B. Zumino, Nucl. Phys. B 47 (1972) 99.

[8] J. Wess and B. Zumino, Nucl. Phys. B 70 (1974) 39.
B. Zumino, Report to the 17th Int. Conf. on high-energy physics
LONDON (1974).

[9] P. Fayet and J. Iliopoulos, Phys. Letters 51 B (1974) 461.

[10] P. Fayet, Nucl. Phys. B 90 (1975) 104.

[11] S.L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. D2 (1970)
1285.

[12] C. Bouchiat, J. Iliopoulos and Ph. Meyer, Phys. Letters 38 B
(1972) 519.

[13] M. K. Caillard and B. W. Lee, Phys. Rev. Letters 33 (1974) 108.
G. Altarelli and L. Maiani, Phys. Letters 52 B (1974) 351.
K. Wilson, Phys. Rev. 179 (1969) 1499.
M. K. Gaillard, B. W. Lee and J. Rosner, Fermi Lab. Pub. 74-34
THY (1974).

[14] P. Fayet, Phys. Letters 48 B (1974) 145.

[15] P. Fayet, Nucl. Phys. B 78 (1974) 14.

[16] M. Y. Han and Y. Nambu, Phys. Rev. 139 (1965) B 1006.

[17] M. A. B. Bég and A. Zee, Phys. Rev. Letters 30 (1973) 675.

[18] H. Fritzsch, M. Gell-Mann and P. Minkowski, Preprint CALT 68-517 (1975).
F. Wilczek, A. Zee, R.L. Kingsley and S.B. Treiman, Preprint Fermi Lab. Pub. 75-44 THY (1975).
R. L. Kingsley, F. Wilczek and A. Zee, Princeton Preprint (1975)

[19] G. J. Feldman and M. L. Perl, Physics Reports 19 C (1975) 5.

[20] R. M. Barnett, Phys. Rev. Letters 34 (1975) 41.
R. M. Barnett, Phys. Rev. D 11 (1975) 3246.

[21] M. A. and C. Bouchiat, Phys. Letters 48 B (1974) 111.

[22] J. Wess and B. Zumino, Nucl. Phys. B 78 (1974) 1.

[23] A. Salam and J. Strathdee, Nucl. Phys. B 76 (1974) 477.
S. Ferrara, J. Wess and B. Zumino, Phys. Letters 51 B (1974) 239

[24] J. Wess and B.Zumino, Phys. Letters 49 B (1974) 52.

[25] J. Iliopoulos and B. Zumino, Nucl. Phys. B 76 (1974) 310.

[26] A. Salam and J. Strathdee, Phys. Letters 49 B (1974) 465.

[27] S. Ferrara and B. Zumino, Nucl. Phys. B 79 (1974) 413.
A. Salam and J. Strathdee, Phys. Letters 51 B (1974) 353.

[28] P. Fayet, Physics Letters 58 B (1975) 67.

[29] P. Fayet, Preprint PTENS 75/5.

[30] S. Ferrara and E. Remiddi, Phys. Letters 53 B (1974) 347.

[31] B. Zumino, Nucl. Phys. B 89 (1975) 535.

[32] W. A. Bardeen, to be published.
B. de Wit and D. Z. Freedman, Phys. Rev. Letters 35 (1975) 827.

LISTE DES ARTICLES JOINTS A LA THESE

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- $|\Delta\vec{I}|$ = 1/2 Rule in Non-Leptonic Weak Interactions with Han-Nambu Quarks. Physics Letters 48B,(1974), 145.
- A Gauge Theory of Weak and Electromagnetic Interactions with Spontaneous Parity-Breaking. Nuclear Physics B78,(1974), 14.
- Introduction to Weak Neutral Currents. Proceedings of the IXth Rencontre de Moriond, Tran Thanh Van ed.,(1974), 87.
- Spontaneously-Broken Supergauge Symmetries and Goldstone Spinors (en collaboration avec J. Iliopoulos). Physics Letters 51B,(1974), 461.
- Supergauge-Invariant Extension of the Higgs Mechanism and a Model for the Electron and its Neutrino. Nuclear Physics B90, (1975), 104.
- Spontaneous Supersymmetry Breaking without Gauge Invariance. Physics Letters 58B,(1975),67.
- Higgs Model and Supersymmetry. Preprint PTENS 75/5.

$|\Delta I| = \frac{1}{2}$ RULE IN NON-LEPTONIC WEAK INTERACTIONS WITH HAN–NAMBU QUARKS

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Gauge theories of weak and electromagnetic interactions involving Han–Nambu quarks are constructed with the following features: small $K_L \to \mu\bar{\mu}$ amplitude and $K_L - K_S$ mass-splitting; a non-leptonic effective Lagrangian $\Delta Y = \pm 1$ leading to the usual soft-pion theorems and obeying a fundamental $|\Delta I| = 1/2$ rule, with corrections of order $\theta^2_{\text{Cabibbo}}$.

The extension to hadrons of renormalizable theories of weak and electromagnetic interactions generally requiring the introduction of "charmed" hadronic states can be made in many ways. Strong constraints come from the selection rules of weak interactions. One of them, the non-leptonic $|\Delta I| = \frac{1}{2}$ rule, does not usually come out automatically in any of the models proposed so far, and one has often to invoque various dynamical built-in mechanisms. Here we want to propose a model with a fundamental $|\Delta I| = \frac{1}{2}$ rule. The rule is exact in the limit of vanishing Cabibbo angle θ_c; the residual $|\Delta I| = \frac{3}{2}$ amplitude, proportional to θ_c^2, can account for the relatively large deviations to the $|\Delta I| = \frac{1}{2}$ rule observed in $K \to 2\pi$ and $K \to 3\pi$ decays.

We shall especially be interested in the SO(3) gauge theory of Georgi and Glashow [1], which has no neutral current. But the hadronic model proposed can be adopted very easily to models involving neutral currents, such as the SU(2) × U(1) gauge theory of Lee, Prentki and Zumino [2]. Let us recall the difficulties encountered when extending the Georgi–Glashow theory to hadrons. The introduction of "charmed" states is always necessary. A first version of the model, with five quarks, has been eliminated [3] because second order weak interactions led to too great a rate for the $K_L \to \mu\bar{\mu}$ decay. In an eight-quark version, the second order weak-interaction difficulties (in the $K_L \to \mu\bar{\mu}$ decay, or the $K_L - K_S$ mass-splitting) are avoided by the procedure of Glashow, Iliopoulos and Maiani [4]; but the role played by these quarks in the strong interaction theory is not clear, and the successful predictions of the naive quark model are lost.

From this point of view the use of the nine Han–Nambu [5] quarks, proposed by Lipkin [6], is more appropriate: it accounts for the low-lying hadron spectrum, gives the predictions of the naive quark model and is compatible with the Fermi statistics for quarks; the model is anomaly-free, and the correct $\pi^\circ \to \gamma\gamma$ amplitude is obtained. The Han–Nambu quarks are denoted by

$$\begin{bmatrix} p_1 & p_2 & p_3 \\ n_1 & n_2 & n_3 \\ \lambda_1 & \lambda_2 & \lambda_3 \end{bmatrix},$$

with integer electric charges

$$\begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}.$$

Each column is a triplet under conventional strong interaction SU(3) group, each row is a triplet under another group, SU(3)′, assumed to be an exact symmetry of strong interactions. Observed hadrons are singlets under SU(3)′; non singlet states are called "charmed".

Using these quarks, Georgi and Glashow [7] constructed a model, equivalent to the Cabibbo theory for semileptonic decays of physical hadrons, without order G or $G\alpha$ contribution to the amplitudes of the $K_L \to \mu\bar{\mu}$ decay and $K\bar{K}$ transition. But this model does not give the well-known current-algebra relations between the non-leptonic decay amplitudes of strange hadrons, such as $K \to 2\pi$ and $K \to 3\pi$; furthermore the model of ref. [7] has been obtained in a quite artificial way, setting some unpleasant amplitudes to zero by violent means.

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A substantial progress has recently been made by Love, Nanopoulos and Ross [8]†, who proposed a very elegant model, depending only on two parameters: the W mass and the Cabibbo angle; all the above constraints are automatically satisfied. However, this model, although very attractive, has no fundamental $|\Delta I| = \frac{1}{2}$ rule.

In order to have such a rule, one may invoque [9] a hypothetical dominance of the diagrams where a Higgs scalar is exchanged; alternately one may double the number of charged gauge bosons [10] in such a way that the order G non-leptonic decays of strange hadrons are due only to neutral currents; both [9] and [10] use eight-quark models. We will show that the $|\Delta I| = \frac{1}{2}$ rule can be implemented in a simple way, in a SO(3) gauge theory (with a single $W^{\pm}$ pair and no neutral current) with Han–Nambu quarks.

We assign left-handed and right-handed quark fields to the SO(3) representations as follows: four gauge triplets:

$$[p_2 \cos\psi - p_3 \sin\psi,\ p_1 \cos\varphi + \lambda_2 \sin\varphi,\ n_1]_L,$$
$$[p_2, n_3, n_1]_R,$$

$$[p_2 \sin\psi + p_3 \cos\psi,\ -p_1 \sin\varphi + \lambda_2 \cos\varphi,\ \lambda_1]_L,$$
$$[p_3, \lambda_2, \lambda_1]_R,$$

and six gauge singlets, which are the orthogonal combinations of neutral fields. In semileptonic decays of strange hadrons, only appears the $SU(3)'$ singlet part (this is denoted by the letter "S") of the charged current:

$$(J_-^{\mu})_S = \tfrac{1}{3} \sum_{i=1}^{3} [\bar{p}_{iL} \gamma^{\mu} n_{iL} \cos\varphi + \bar{p}_{iL} \gamma^{\mu} \lambda_{iL} (\sin(\varphi + \psi) - \sin\varphi)] .$$

For these decays the theory is equivalent to the Cabibbo theory. The Cabibbo angle θ_c is given by $\tan\theta_c = [\sin(\varphi + \psi) - \sin\varphi]/\cos\varphi$; we have $\theta_c \approx \psi$ for small values of ψ. The angle β introduced by Georgi and Glashow is determined by lepton-hadron universality: $\sin\beta = \cos\varphi/3\cos\theta_c$; the intermediate boson mass is $M_W \approx [53\ \mathrm{GeV}/c^2]\ \sin\beta \approx [18\ \mathrm{GeV}/c^2]\ \cos\varphi$.

† Being unaware of the recent work of A. Love, D.V. Nanopoulos and G.G. Ross, we have discovered the same model independently.

We examine now the amplitudes of the $K\bar{K}$ transition (or other $\Delta Y = \pm 2$ processes), and the $K_L \to \mu\bar{\mu}$ decay. Here we shall not be interested in the diagrams with Higgs scalars: their contributions are small if these scalars have high enough masses.

The exchange of a W between two quark lines cannot contribute to the $K\bar{K}$ transition amplitude, which is zero at order G. It is also zero at order $G\alpha$ because the exchange of two W's between two quark lines does not contribute in the limit of massless quarks; these diagrams give an amplitude proportional to $G\alpha\, m^2/M_W^2$, where m is a quark mass, which cannot therefore be too large. We can also verify that there is no $G\alpha^2$ term. Thus one explains the small value of the $K_L - K_S$ mass-splitting.

In a similar way, no order-$G\alpha$ contribution appears in the $K_L \to \mu\bar{\mu}$ decay amplitude. However, the exchange of two W's between a quark line $(\lambda_1, \lambda_2, n_1)$ and a lepton line (μ, ν', μ) gives an amplitude proportional to $G\alpha\, m m_{\nu'}/M_W^2$; therefore the mass $m_{\nu'}$ of the heavy neutrino cannot be too large; a few GeV/c^2 would be a reasonable value. Then the $K_L \to \mu\bar{\mu}$ amplitude is of order $G\alpha^2$, in agreement with the experimental results: there is of course, in this model as in any model, a $K_L \to \mu\bar{\mu}$ decay amplitude proportional to $G\alpha^2$, due to the $K_L \to \gamma\gamma \to \mu\bar{\mu}$ processes.

For semileptonic decays where the lepton pair can be produced in a $J \neq 0$ state, such as $K \to \pi e\bar{e}$, there is an order $G\alpha$ amplitude, as required by unitarity, coming from the exchange of a photon (and not of a W pair) between the lepton and hadron lines.

We now come to the non-leptonic decays of strange hadrons, which are $SU(3)'$ singlets. These decays are described, in the local limit, by the effective Lagrangian density

$$L = \frac{G}{\sqrt{2}} \frac{4}{\sin^2\beta} (: J_{\mu -} J_+^{\mu} :)_{S, \Delta Y = \pm 1} .$$

The only right-handed fields present in this expression are the zero isospin fields λ_{1R} and λ_{2R}. Therefore L commutes with $F_K + F_K^5(t)$ where the F_K are the isospin generators, and the $F_K^5(t)$ the axial operators associated. With PCAC and current-algebra, we obtain the usual relations between decay amplitudes, for example $K \to 2\pi$ and $K \to 3\pi$.

L contains a term proportional to $(: \bar{\lambda}_{2L} \gamma^{\mu} n_{1L}\, \bar{\lambda}_{1R} \gamma_{\mu} \lambda_{2R} + \mathrm{h.c.} :)_S$, which induces

$|\Delta I| = \frac{1}{2}$ transitions only, and a term involving left currents only, denoted by L_L. We define the SU(3)′ singlet currents $\bar{p}\gamma^\mu n = \Sigma_{i=1}^{3} \bar{p}_i \gamma^\mu n_i$, etc., which have the same SU(3) transformation properties and commutation relations as in the usual quark model. Using a Fierz transformation, we can re-write L_L with these currents only, and find:

$$L_{L,\Delta Y=-1} = \frac{3G}{2\sqrt{2}} \frac{\cos^2\theta_c}{\cos^2\varphi}$$
$$\times \{[3\sin(2\varphi+\psi) - \sin 2\varphi]\bar{\lambda}_L \gamma^\mu p_L \bar{p}_L \gamma_\mu n_L$$
$$- [\sin(2\varphi+\psi) + \sin 2\varphi]\bar{\lambda}_L \gamma^\mu n_L \bar{p}_L \gamma_\mu p_L$$
$$+ [\sin 2\varphi]\bar{\lambda}_L \gamma^\mu n_L \bar{\lambda}_L \gamma_\mu \lambda_L\}. \quad (1)$$

We note that this effective Lagrangian density is finite when θ_c goes to zero, in contrast with the conventional current-current theory where a factor $\sin\theta_c$ is present. This is in fact due to the product of the "charmed" currents which does not vanish with θ_c.

The $|\Delta I| = \frac{3}{2}$ amplitude is proportional to $[\sin(2\varphi+\psi) - \sin 2\varphi]$ and can be made to vanish. Moreover, for any value of φ, the $|\Delta I| = \frac{1}{2}$ rule is obtained in the limit $\theta_c \to 0$: the $|\Delta I| = \frac{1}{2}$ amplitude is then finite, and the $|\Delta I| = \frac{3}{2}$ amplitude goes to zero, like θ_c if $\cos 2\varphi \neq 0$; then the latter amplitude has the same order of magnitude as in the usual current-current model.

The most interesting model is obtained for $\cos 2\varphi = 0$, which fixes the intermediate boson mass to the relatively low value $M_W = 13\ \mathrm{GeV}/c^2$. Taking $\varphi = \frac{1}{4}\pi$ we find that the choice $\varphi + \psi = \frac{1}{3}\pi$ yields a value of the Cabibbo angle given by $\tan\theta_c = \sqrt{\frac{3}{2}} - 1 \approx 0.225$. The $|\Delta I| = \frac{3}{2}$ amplitude is proportional to $(1 - \cos\psi) \approx \frac{1}{2}\theta^2$; it is non-zero but small enough, and can account for the observed deviations from the $|\Delta I| = \frac{1}{2}$ rule. Thus, the existence of this residual $|\Delta I| = \frac{3}{2}$ amplitude in this model appears to be strongly connected with the non-zero value of the Cabibbo angle.

For the deep inelastic scattering, the predictions of the model are the same as those of the three-quark or three × three quark models (Han–Nambu or color quarks) below the charm threshold; above it, they are distinguishable. Notice also that the absence of the factor $\sin^2\theta_c$ in the product of SU(3)′ octet parts of $\Delta Y = +1$ and $\Delta Y = -1$ currents can account for some large parity violation effects occurring in nuclear physics.

This hadronic model can be adapted very easily to models involving neutral currents, such as the SU(2) × U(1) model of Lee, Prentki and Zumino [2]. However, in this model, in order to maintain lepton-hadron universality it is necessary to replace – as in [8] – ν by $\nu\sin\beta + \nu'\cos\beta$ where ν' is a heavy neutrino. The left-handed fields are assigned to the SU(2) representations as in the previous model, but all right-handed fields are SU(2) singlets. Let θ_W by the Weinberg angle. The neutral gauge boson Z_o, coupled to the current $J_z^\mu = J_3^\mu - 2\sin^2\theta_W J_{em}^\mu$, has only diagonal couplings. Z_o does not enter the discussion on the $K\bar{K}$ transition and the $K_L \to \mu\bar{\mu}$ decay amplitudes, which are respectively of order $G\alpha m^2/M_{W^2}$ and $G\alpha^2$, (m is a quark mass). The current algebra relations between decay amplitudes such as $K \to 2\pi$ and $K \to 3\pi$ are trivial here, and expression (1) is now the complete effective Lagrangian density $L_{\Delta Y=-1}$ giving – as before – the non-leptonic $|\Delta I| = \frac{1}{2}$ rule. The W mass is $M_W = [53\ \mathrm{GeV}/c^2]\sin\beta/\sin\theta_W$, which is $[13\ \mathrm{GeV}/c^2]/\sin\theta_W$ for $\varphi = \frac{\pi}{4}$.

In conclusion, we have presented models which account for the selection rules of weak interactions, including a fundamental non-leptonic $|\Delta I| = \frac{1}{2}$ rule, obtained in the limit of vanishing Cabibbo angle, with violations $\sim \theta_c^2$.

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[1] H. Georgi and S.L. Glashow, Phys. Rev. Letters 28 (1972) 1494.
[2] B.W. Lee, Phys. Rev. D6 (1972) 1188.
J. Prentki and B. Zumino, Nucl. Phys. B47 (1972) 99.
[3] B.W. Lee, J. Primack and S. Treiman, Phys. Rev. D7 (1973) 510.
[4] S.L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. D2 (1970) 1295.
[5] M.Y. Han and Y. Nambu, Phys. Rev. 139 (1965) B1006.
[6] H.J. Lipkin, report NAL THY 85 (1972).
[7] H. Georgi and S.L. Glashow, Phys. Rev. D7 (1973) 561.
[8] A. Love, D.V. Nanopoulos and G.G. Ross, Nucl. Phys. B55 (1973) 33.
[9] B.W. Lee and S.B. Treiman, Phys. Rev. D7 (1973) 1211.
[10] A. Pais, report COO-22 32 B-7, Rockefeller University, N.Y. (1973).

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A GAUGE THEORY OF WEAK AND ELECTROMAGNETIC INTERACTIONS WITH SPONTANEOUS PARITY BREAKING

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Abstract: We present a unified gauge theory of weak and electromagnetic interactions in which parity is spontaneously broken together with gauge invariance, by the Higgs mechanism. The gauge group is SU(2) × U(1), and a heavy neutrino is associated with every charged lepton. After the breaking of the original parity-conserving theory, both a purely vector electromagnetic current and the usual V-A charged currents are obtained. Z is coupled to a vector electron current, and the model predicts equal ν_μe and $\bar{\nu}_\mu$e cross sections. Extension to hadrons is made by introducing three charmed quarks p′, n′ and λ′ of the same charges as p, n and λ. All the experimental results (ν_μe, $\bar{\nu}_\mu$e, $\bar{\nu}_e$e, ν_μ and $\bar{\nu}_\mu$ hadron scatterings) are compatible with a value of $\sin^2\theta_W$ of order 0.4.

1. Introduction

We know that weak and electromagnetic interactions can be included in a unified renormalizable gauge theory. For this purpose, one has to introduce new objects, "neutral currents" [1, 3] or "heavy leptons" [2, 3], which have actively been searched experimentally. Neutral currents have been observed in the ν_μ and $\bar{\nu}_\mu$ hadron collisions [4].

The most classical model with neutral currents is the Weinberg model [1], in which no new lepton has been introduced. The gauge group is SU(2) × U(1), and there is only one vector boson other than the photon, Z. But in this model, which is the simplest in which neutral currents appear, the right-hand side and the left-hand side are treated in extremely different ways. A satisfying unified theory of weak and electromagnetic interactions should be able to account for the simultaneous existence of a purely vector electromagnetic current and of V-A charged currents in a natural way.

We introduce a new lepton, the heavy neutrino ν', and keep the same SU(2) × U(1) gauge group as in the Weinberg model. The original theory is parity invariant, and parity and gauge invariance are spontaneously broken at the same time, by the Higgs

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mechanism. Parity is violated in a maximal way in the classical weak interactions: only the left component of the massless neutrino field, ν, appears in the charged currents; the right-handed charged currents couple the electron only to the heavy neutrino, ν'. The couplings of the electron with both the neutral vector bosons γ and Z are parity conserving: then the parity-invariance of the electromagnetic interaction comes out naturally. One of the most important consequences of the model is that the ν_μe and $\bar{\nu}_\mu$e cross sections are equal. This typical feature will allow the model to be tested, as soon as the statistics are sufficient. These cross sections (at order G) have no lower bound, and vanish for $\theta_W = \frac{1}{4}\pi$. One also finds well-defined predictions for the $\bar{\nu}_e$e cross section, but the flux determination is difficult in the corresponding experiment. All these results are compatible, and restrict $\sin^2\theta_W$ to the interval 0.17–0.70.

An amusing remark is that the right-handed neutrino fields, though not appearing in the charged currents, are coupled with Higgs scalars; so neutrinos of unusual polarization can be produced in physical decays, but only in pairs. The study of μ decay gives a lower mass limit, not a very constraining one, on the charged Higgs boson. We have also verified that the weak corrections to the anomalous magnetic moment of the muon and the electron are very small, and cannot at present disagree with experiment.

Finally, the model can be extended to hadrons by introducing three charmed quarks p′, n′ and λ′ that have the same charges as p, n and λ. With a naive quark parton model, we compute, in terms of θ_W, the ratios

$$R = \frac{\sigma^{NC}_{\nu\text{-nucleon}}}{\sigma^{CC}_{\nu\text{-nucleon}}}, \qquad \bar{R} = \frac{\sigma^{NC}_{\bar{\nu}\text{-nucleon}}}{\sigma^{CC}_{\bar{\nu}\text{-nucleon}}}.$$

The comparison with the Cern Gargamelle experiments shows that these results are compatible, with $\sin^2\theta_W$ of order 0.4 [5].

We now construct the model explicitly.

2. Construction of the model

The gauge group is SU(2) × U(1), as in the Weinberg model [1]. The gauge bosons W^μ and B^μ are coupled to T and F, the SU(2) × U(1) generators, respectively. The local gauge transformations are such that the covariant derivative is defined by:

$$iD_\mu = i\partial_\mu - gT\cdot W_\mu + \tfrac{1}{2}g'FB_\mu .$$

There are three leptons in the model: electron, neutrino and heavy neutrino (and three more when the μ is introduced). They are distributed in a SU(2) × U(1) singlet, S, and a SU(2) doublet, D, which has the U(1) quantum number $F = 1$, and will be written

$$\begin{bmatrix} N \\ e \end{bmatrix},$$

when the spontaneous gauge symmetry breaking has specified the direction of the charge.

The neutral field N, which will hereafter be expressed in terms of the two ν and ν' neutrino fields, must not be coupled to the photon γ but only to the neutral vector boson Z. This allows us to define Z and γ from a rotation of W_μ^3 and B_μ. Let us write $\mathrm{tg}\,\theta_W = g'/g$. We have:

$$Z_\mu = \cos\theta_W W_\mu^3 - \sin\theta_W B_\mu\,, \quad A_\mu = \sin\theta_W W_\mu^3 + \cos\theta_W B_\mu\,.$$

The Z and γ couplings are fixed by the Lagrangian density:

$$\mathcal{L} = -\tfrac{1}{2}\bar{N}\gamma^\mu N[g W_\mu^3 - g' B_\mu] + \tfrac{1}{2}\bar{e}\gamma^\mu \mathrm{e}\,[g W_\mu^3 + g' B_\mu]$$

$$= -\tfrac{1}{2}\sqrt{g^2+g'^2}\,\bar{N}\gamma^\mu N Z_\mu + \tfrac{1}{2}\sqrt{g^2+g'^2}\,\bar{e}\gamma^\mu \mathrm{e}$$

$$\times\,[2\sin\theta_W\cos\theta_W A_\mu + (\cos^2\theta_W - \sin^2\theta_W) Z_\mu]\,.$$

We call

$$e = \sqrt{g^2+g'^2}\,\sin\theta_W\cos\theta_W = g\sin\theta_W = g'\cos\theta_W$$

which can be supposed positive. The electron charge is $(-\mathrm{e})$.

Both γ and Z couplings to the electron are parity-conserving; this last coupling vanishes for $\theta_W = \frac{1}{4}\pi$.

2.1. *Spontaneous breaking of gauge invariance and parity*

Now we show how the Higgs mechanism gives such a breaking. We must use Higgs doublets, in order to couple the S and D fields, and exclude other Higgs fields, such as singlets or triplets, for simplicity.

The electron mass must already exist before the breaking and then the Lagrangian density contains a term $-m_\mathrm{e}\bar{D}D$. But it should not contain a term $-m_\mathrm{s}\bar{S}S$: indeed, when the vacuum mean values of the Higgs fields are going to zero, the broken theory follows a continuous path to the unbroken theory; the former contains a massless particle, the neutrino; then, in the latter also we have a massless particle which is necessarily S.

We are led to impose the invariance with respect to the group of Q transformations to the theory *:

$$Q D Q^{-1} = D\,,\; Q S Q^{-1} = \mathrm{e}^{i\gamma_5\alpha} S\,.$$

Let us call φ' and φ'' the Higgs doublets coupled to $\bar{D}_\mathrm{R} S_\mathrm{L}$ and $\bar{D}_\mathrm{L} S_\mathrm{R}$; at this moment, we do not suppose they are orthogonal, or even independent. φ' and φ'' behave like D under the gauge group, and they are complex. The parity invariance of the unbroken theory shows that φ' and φ'' must be exchanged by parity, and the Lagrangian density contains the terms:

$$\mathcal{L} = \ldots - m_\mathrm{e}\bar{D}D + h_\mathrm{e}\,[\bar{D}_\mathrm{R} S_\mathrm{L}\varphi' + \bar{D}_\mathrm{L} S_\mathrm{R}\varphi'' + \mathrm{h.c.}]\,.$$

The Q invariance of this part of $\mathcal{L}$ defines the transformation laws of φ' and φ'' under Q:

$$Q\varphi' Q^{-1} = \mathrm{e}^{-i\alpha}\varphi'\,,\quad Q\varphi'' Q^{-1} = \mathrm{e}^{i\alpha}\varphi''\,.$$

This shows that φ' and φ'' are orthogonal fields; it is impossible to use only one complex Higgs doublet; two at least are necessary: the orthogonal doublets φ' and φ'' exchanged by parity *.

We have also in the Lagrangian density the self-coupling terms of the Higgs doublets, which are written in the appendix; we show that, under some inequalities constraining the coupling constants, one, and only one, of the two fields φ' and φ'' takes a non-zero vacuum mean value. From this follows parity violation **. We choose $\langle\varphi'\rangle \neq 0$ and $\langle\varphi''\rangle = 0$, in order to have a left-handed polarization for the usual neutrino. We choose the gauge in order to have $\varphi'^{\pm} = 0$ and φ'^0 Hermitian, with a positive vacuum mean value v. Finally, in the theory, we have a charged Higgs field $\varphi''^{\pm}$, and two neutral ones φ''^0 complex and φ'^0 Hermitian.

After the breaking we have the following mass terms in the Lagrangian density:

$$-m_\mathrm{e}\bar{e}\,e - \{\bar{N}_\mathrm{R}(m_\mathrm{e}N_\mathrm{L} - h_\mathrm{e}vS_\mathrm{L}) + (m_\mathrm{e}\bar{N}_\mathrm{L} - h_\mathrm{e}v\bar{S}_\mathrm{L})N_\mathrm{R}\}\,.$$

S_R does not appear in these mass terms, and is identified to ν_R. ν is massless, at least at the zeroth order. To show that its mass is identically zero, we just note that the broken theory is invariant under a change of sign of both S_R and φ''.

N_R is identified with ν'_R. We write

$$m_\mathrm{e} = m_{\nu'\mathrm{e}}\cos\beta_\mathrm{e}\,,\quad h_\mathrm{e}v = m_{\nu'\mathrm{e}}\sin\beta_\mathrm{e}\,.$$

m_e is the electron mass, and $m_{\nu'\mathrm{e}}$ the heavy neutrino mass with:

$$\nu'_\mathrm{L} = N_\mathrm{L}\cos\beta - S_\mathrm{L}\sin\beta\,,\quad \nu_\mathrm{L} = N_\mathrm{L}\sin\beta + S_\mathrm{L}\cos\beta\,.$$

We can now rewrite the gauge doublet and singlet in terms of the physical fields:

$$D = \begin{bmatrix} (\nu'_\mathrm{L}\cos\beta + \nu_\mathrm{L}\sin\beta) + \nu'_\mathrm{R} \\ \mathrm{e} \end{bmatrix}\,,\quad S = (-\nu'_\mathrm{L}\sin\beta + \nu_\mathrm{L}\cos\beta) + \nu_\mathrm{R}\,.$$

* It is also possible to consider a discrete symmetry group instead of a continuous one, taking $\alpha = \frac{1}{2}n\pi$. The group has only four elements, acting on S as ± 1 and $\pm i\gamma_5$. We then have an extra term, gauge, parity, and Q-invariant, proportional to $(\varphi'^+\varphi''\varphi'^+\varphi'' + \varphi''^+\varphi'\varphi''^+\varphi')$ in the Lagrangian density in addition to the terms written in the appendix. It is very easy to take this term into account in the calculations of the appendix.

* We will limit ourselves to the Higgs doublets φ' and φ'': it is useless to consider extra doublets, which would have no coupling with the fermion fields.

** This parity violation is "maximal". On the contrary, in the absence of the Q invariance restricting the self-couplings of the Higgs doublets, we would generally have a non-maximal parity-violation: $|\langle\varphi'\rangle|$ and $|\langle\varphi''\rangle|$ are different, but none is zero, and then the theory contains no massless fermion.

Let us note that the expressions are analogous with those of the Georgi-Glashow model [2]. Here we also use a heavy neutrino, but no heavy electron. Such a repartition of e, ν and ν' in a gauge doublet and a gauge singlet had already been written by Beg and Zee [6], but with $\sin\beta = \frac{1}{3}$ for lepton-hadron universality.

The masses of the intermediate bosons $W^{\pm}$ and Z are given by

$$M_W^2 = \tfrac{1}{2} g^2 v^2 \ , \quad M_Z^2 = \tfrac{1}{2}(g^2 + g'^2) v^2 \ .$$

The Fermi constant is fixed by

$$\frac{G}{\sqrt{2}} = \frac{g^2}{8M_W^2} \sin\beta_e \sin\beta_\mu \simeq \frac{g^2}{8M_W^2} \ ,$$

since the zeroth order mass relations

$$m_e = m_{\nu'_e} \cos\beta_e \ , \quad m_\mu = m_{\nu'_\mu} \cos\beta_\mu$$

show that the angles β_e and β_μ are near $\frac{1}{2}\pi$. We find then:

$$M_W = 37 \text{ GeV}/c^2 \ \frac{1}{\sin\theta_W} \ , \quad M_Z = 37 \text{ GeV}/c^2 \ \frac{1}{\sin\theta_W \cos\theta_W} \ .$$

The photon, of course, is massless.

It would be very attractive to take $\beta_e = \frac{1}{2}\pi$ (then the zeroth order electron mass vanishes), and to obtain from higher order corrections a finite mass for the electron. This is impossible for symmetry reasons: when β_e is $\frac{1}{2}\pi$, the term $-m_e \bar{D} D$ is the only one to connect ν_L and ν'_R, and must be zero, not only at the zeroth order but identically. The transformation $D_L \to -D_L$, $\varphi'' \to -\varphi''$ is a symmetry of the theory, from which the vanishing of the electron mass at all orders follows. Hence we cannot hope to determine the electron mass in this way.

2.2. *e-μ universality*

The study of the muon is exactly similar to that of the electron. It is possible, if one so wishes, to put in an exact e-μ universality by hand, by fixing $\beta_e = \beta_\mu$. One generally acts in this way in models including heavy leptons, such as the Georgi-Glashow one. But it seems rather arbitrary.

Then, we find the mass relation $m_e/m_{\nu'e} = m_\mu/m_{\nu'_\mu}$ which makes the ν'_μ very heavy, at least 100 GeV/c^2. But it does not lead to disagreements with experiment, either in μ decay or in g-2.

Finally, notice that it is at least as satisfactory not to impose $\beta_e = \beta_\mu$. The e-μ universality is then naturally obtained, but only approximately. This is however quite sufficient: as soon as ν'_μ is heavier than 2 GeV/c^2, we have $\cos\beta_\mu \lesssim \frac{1}{20}$ (with $\cos\beta_e \simeq 0$), and the corrections to e-μ universality are at most of the order of some ‰.

3. Neutrino electron scattering

In terms of the angle θ_W, such a model gives well-defined predictions for the purely leptonic transition amplitudes.

Let us recall the experimental situation for the $\nu_\mu e$ and $\bar{\nu}_\mu e$ diffusions. The first runs gave no ν_μ event and one $\bar{\nu}_\mu$ event, which has only a small probability to be due to the background. From this one could infer upper limits on the cross sections [7]:

$$\sigma_{\nu_\mu e} < 0.26\ 10^{-41} \text{cm}^2\, E\,(\text{GeV}) \ , \quad \sigma_{\bar{\nu}_\mu e} < 0.88\ 10^{-41}\ \text{cm}^2\, E\,(\text{GeV}) \ .$$

In the absence of neutral currents, such processes are possible only at order $G\alpha$, as two charged currents are exchanged. On the contrary in models including neutral currents, the scattering is allowed at order G, by the exchange of the vector boson Z. One of the characteristics of the proposed model is that the coupling of Z with the electron is parity-conserving, hence at order G the equality of the $\nu_\mu e$ and $\bar{\nu}_\mu e$ cross sections.

Let us write, in the local limit and with the approximation $\sin\beta_e \simeq \sin\beta_\mu \simeq 1$, the effective Lagrangian density responsible for this process, at order G:

$$\mathcal{L}^{\text{eff}} = -\frac{g^2 + g'^2}{4M_Z^2} \bar{\nu}_{\mu L} \gamma^\mu \nu_{\mu L} \ \bar{e} \gamma_\mu e \ (\cos^2\theta_W - \sin^2\theta_W)$$

$$= \frac{G}{\sqrt{2}} \bar{\nu}_\mu \gamma^\mu (1 + \gamma_5) \nu_\mu \ \bar{e} \gamma_\mu e \ (2\sin^2\theta_W - 1) \ .$$

With the usual notations, we have:

$$g_V = 2\sin^2\theta_W - 1 \ , \quad g_A = 0 \ .$$

$g_V + g_A$ has the same value as in the Weinberg model, since the couplings of Z with the left-handed electron current $\bar{e}_L \gamma^\mu e_L$ are alike in both models.

Neglecting the electron mass, we write the cross sections:

$$\sigma_{\nu_\mu e} = \sigma_{\bar{\nu}_\mu e} = \sigma_0 g_V^2 = 0.56\ 10^{-41}\ (\text{cm}^2)\, E(\text{GeV})\, (2\sin^2\theta_W - 1)^2 \ ,$$

where σ_0 stands for $\sigma_{\bar{\nu}_e e}^{V-A}$. The upper bound on $\sigma_{\nu_\mu e}$ gives then:

$$0.16 < \sin^2\theta_W < 0.84 \ .$$

We compare on the same figure the predictions of this model with those of Weinberg's for which

$$g_V = 2\sin^2\theta_W - \tfrac{1}{2}, \quad g_A = -\tfrac{1}{2},$$

(see fig. 1). All these cross sections are equal for $\sin^2\theta_W = \frac{1}{4}$, a value for which the coupling of Z to the electron in the Weinberg model is purely axial. The new results expected for the $\bar{\nu}_\mu$ runs should soon allow the determination of θ_W to be improved.

We now come to the $\bar{\nu}_e e$ scattering, which has been studied at the Savannah River

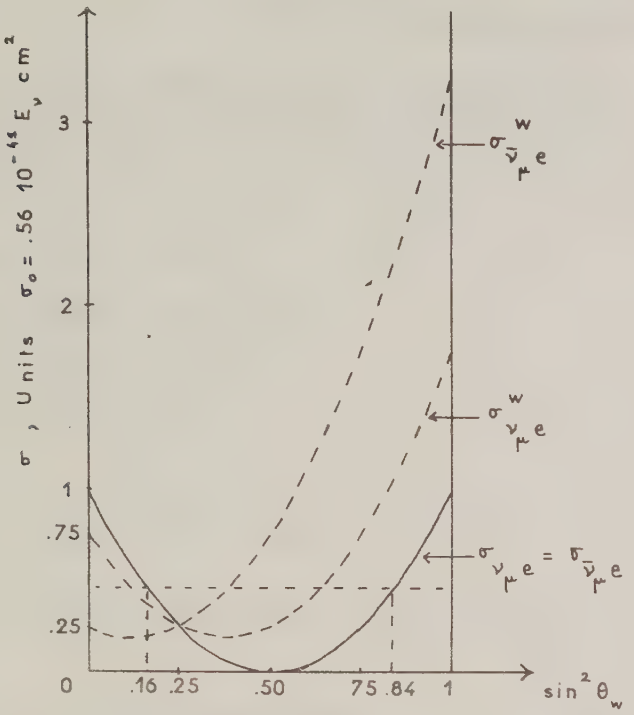

Fig. 1. Variation with $\sin^2\theta_W$ of $\nu_\mu e$ and $\bar{\nu}_\mu e$ cross sections in the proposed model and in the Weinberg model (index W).

reactor [8]. The amplitude, allowed at order G in the V-A theory, is modified by the presence of neutral currents. The coefficients g_V and g_A are now:

$$g_V = 2\sin^2\theta_W\,,\quad g_A = 1\,.$$

The results of the experiment are unfortunately very much depending on the spectrum which is not measured but only estimated. They are expressed by the inequality:

$$\left(\frac{g_V+g_A}{2}\right)^2 + 14\left(\frac{g_V-g_A}{2}\right)^2 \leqslant 2\,.$$

Such an inequality, which is verified for the V-A theory and gives $\sin^2\theta_W < 0.3$ in the Weinberg model, here gives:

$$0.17 < \sin^2\theta_W < 0.70\,.$$

The model is thus compatible with all the experimental results existing for purely leptonic transitions. The interval of variation of $\sin^2\theta_W$ which was 0.16–0.84 from the absence of $\nu_\mu e$ event, is now reduced to 0.17–0.70, from the $\bar{\nu}_e e$ experiment.

4. μ decay and lepton anomalous magnetic moments

We must verify that the corrections obtained from the model are compatible with the experimental limits.

4.1. μ-decay

The exchange of the charged Higgs boson φ'' contributes to the μ-decay amplitude. In addition to the electron, a ν_μ and a $\bar{\nu}_e$ are produced in this decay, but with the unusual polarizations, respectively right and left. Although they do not appear in the charged or neutral currents, the right-handed neutrinos and left-handed antineutrinos can appear in physical processes, when a Higgs scalar φ'' is exchanged. The invariance of the broken theory under the transformation $\varphi'' \to -\varphi''$, $\nu_R \to -\nu_R$ compels them to be produced only in pairs.

This decay amplitude corresponds, in the local limit, to the effective Lagrangian density:

$$\mathcal{L}_{\text{eff}} = -\frac{h_\mu h_e}{m^2_{\varphi''_\pm}}\,[\bar{\mu}_L \nu_{\mu R}\bar{\nu}_{eR} e_L + \text{h.c.}]$$

$$= -\frac{G}{\sqrt{2}}\,\frac{m_{\nu'_\mu} m_{\nu'_e}}{m^2_{\varphi''_\pm}}\,[\bar{\mu}(1-\gamma_5)\nu_\mu\bar{\nu}_e(1+\gamma_5)e + \text{h.c.}]\,.$$

Instead of $\frac{3}{4}$ in the V-A theory, the Michel parameter is now

$$\rho = \frac{3}{4 + (m_{\nu'_\mu} m_{\nu'_e}/m^2_{\varphi''_\pm})^2}\,.$$

We have $\rho = 0.752 \pm 0.003$, and then find:

$$m_{\varphi''_\pm} > 3.7\sqrt{m_{\nu'_\mu} m_{\nu'_e}}\,.$$

Such a condition on the $\varphi''_\pm$ mass is not very restrictive.

4.2. Corrections to the muon anomalous magnetic moment

We show now that the weak corrections to the anomalous magnetic moment of the muon are negligible. For $a = \frac{1}{2}(g-2)$ the contribution of quantum electrodynamics, computed up to the third order, and the contributions due to strong interactions give the theoretical prediction [9]

$$a_{\text{th}} = 0.001\,165\,901 \pm 0.000\,000\,010\,.$$

The experimental value is [10]

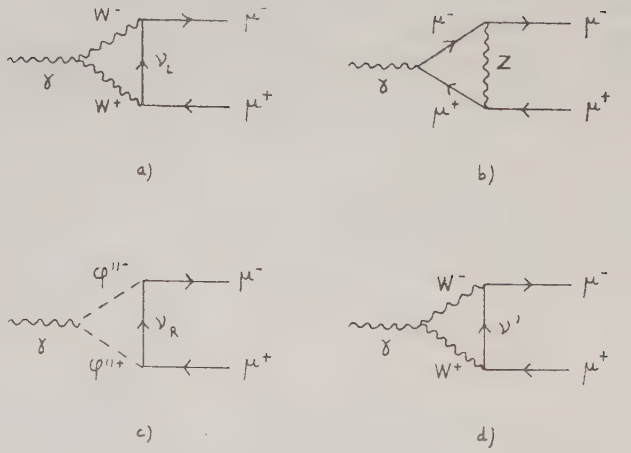

Fig. 2. Diagrams contributing to the muon anomalous magnetic moment.

$$a_{\text{exp}} = 0.001\,166\,160 \pm 0.000\,000\,310 .$$

Thus (with 95% confidence), we can say that any extra contribution to the theoretical value Δa must fall within the limits:

$$-300\ 10^{-9} < \Delta a < 900\ 10^{-9} . \qquad \text{(R)}$$

The diagram [9, 11] (fig. 2a) gives the same contribution as in every gauge theory:

$$a_\nu = \frac{Gm_\mu^2}{8\pi^2\sqrt{2}}\ \frac{10}{3} \simeq 4 \cdot 10^{-9} .$$

The diagram [12] (fig. 2b) gives a contribution (with $m_Z \gg m_\mu$)

$$a_Z = \frac{Gm_\mu^2}{6\pi^2\sqrt{2}}\ (2\sin^2\theta_W - 1)^2 < 1.5 \cdot 10^{-9} .$$

There is a diagram with the charged Higgs bosons (fig. 2c) which contributes for:

$$a_\varphi = -\frac{Gm_\mu^2}{4\pi^2\sqrt{2}} \int_0^1 \frac{z(1-z)\,dz}{m^2_{\varphi''_\pm} - z\,m_\mu^2}\ m^2_{\nu'_\mu} \simeq -\frac{Gm_\mu^2}{24\pi^2\sqrt{2}}\ \frac{m^2_{\nu'_\mu}}{m^2_{\varphi''_\pm}} .$$

Using the lower bound established for $m_{\varphi''_\pm}$, we find:

$$|a_\varphi| < 0.03 \cdot 10^{-9}\ \frac{m_{\nu'_\mu}}{m_{\nu'_e}} .$$

Finally, we have a diagram with the heavy neutrino ν': (fig. 2d). This diagram exists also in the Georgi-Glashow model where, in the limit $m_\mu \ll m_{\nu'_\mu}$, it gives [13] a dominant contribution

$$a_{\nu'} \sim \frac{Gm_\mu^2}{8\pi^2\sqrt{2}}\ \frac{m_{\nu'_\mu}\cos\beta}{m_\mu}$$

In the Georgi-Glashow model, this contribution can be relatively big, because of the inequality

$$2m_{\nu'_\mu}\cos\beta > m_{M_+} .$$

The relation (R) gives then an upper limit to the mass of the heavy muon M_+ and the model may get into trouble if the measurement of the anomalous magnetic moment is improved. On the contrary, nothing like that happens in the proposed model, in which we have $m_{\nu'_\mu}\cos\beta = m_\mu$, which gives for the above diagram a contribution of the order of 10^{-9}. All the weak contributions to $a = \frac{1}{2}(g-2)$ finally are at most of the order of a few 10^{-9}, which is quite below the present experimental precision.

One expects to know the experimental value of a with an uncertainty of the order of $15 \cdot 10^{-9}$ soon (the uncertainty on the strong interactions contribution is of the order of $5 \cdot 10^{-9}$). Although this would be a very notable progress, we cannot yet hope to reach the weak contributions, of the order of a few 10^{-9}; the computation of the fourth order Q.E.D. terms would be necessary at this level, anyhow.

Let us notice however that the present CERN experiment may cause trouble to the Georgi-Glashow model, as it puts a too low higher limit to the heavy muon mass.

The weak contributions to the electron anomalous magnetic moments are of the order of 10^{-13}, an order of magnitude at present inaccessible both to experiment and theory (Q.E.D. computation, estimation of the strong interaction contribution).

5. Extension to hadrons

As in every unified theory of weak and electromagnetic interactions, the introduction of hadrons raises some difficulties, in connection with neutral currents changing strangeness, and with the way the quarks take masses. So, the hadron sector of the model is not so welldefined as the lepton sector.

The hadrons are introduced in analogy with leptons: to the six leptons e, ν_e, ν'_e; μ, ν_μ and ν'_μ, we associate the six quarks p, $n(\theta_C)$ n'; p', $\lambda(\theta_C)$ and λ'. p, n and λ are the usual quarks, θ_C is the Cabibbo angle, and p', n' and λ' are three charmed quarks, of the same charges $\frac{2}{3}$, $-\frac{1}{3}$, $-\frac{1}{3}$, as p, n, and λ. The gauge doublets and singlets are:

$$\left[\begin{matrix} p \\ n_L(\theta_C)\sin\beta + n'_L\cos\beta + n'_R \end{matrix}\right], \quad \left[\begin{matrix} p' \\ \lambda_L(\theta_C)\sin\beta' + \lambda'_L\cos\beta' + \lambda'_R \end{matrix}\right];$$

$$(n_L(\theta_C)\cos\beta - n'_L\sin\beta + n_R(\theta_C)), \quad (\lambda_L(\theta_C)\cos\beta' - \lambda'_L\sin\beta' + \lambda_R(\theta_C)).$$

These SU(2) doublets and singlets have respectively the U(1) quantum numbers $-\frac{1}{3}$ and $\frac{2}{3}$. A difficulty of this model is that the Higgs mechanism, at least as we have exposed it, leaves the n and λ quarks massless.

An approximate lepton-hadron universality is insured by choosing the two angles β and β' near $\frac{1}{2}\pi$ (as it was for β_e and β_μ). The theory is then equivalent to the Cabibbo theory for $\Delta Q = \pm 1$ semileptonic transitions between uncharmed states. The couplings of Z with the uncharmed states conserve strangeness *. The amplitudes of $\Delta Q = 0$ semileptonic decays of strange hadrons, and of the $K\bar{K}$ transition, vanish at order G.

The procedure of Glashow, Iliopoulos and Maiani [14] ensures that, in the limit of equal p and p′ masses, the $K \to \mu\bar{\mu}$, $K \to \pi\nu\bar{\nu}$ and $K \to \bar{K}$ transitions are also forbidden at order $G\alpha$. The $K \to \pi e\bar{e}$ decay is allowed at order $G\alpha$ (as shown by the unitarity relation), but its amplitude is due to the exchange of a γ only, and not of two W's, between the hadron and lepton lines. So, the amplitudes of $\Delta Q = 0$ semileptonic decays of strange hadrons, and the $K_L - K_S$ mass splitting are small enough, in agreement with the experimental results.

We are now interested in the neutrino hadron scattering. Ignoring the charmed quarks, we can describe it, in the local limit and with $\sin\beta \simeq \sin\beta' \simeq 1$, by the effective Lagrangian density

$$\mathcal{L}^{\text{eff}} = \frac{G}{\sqrt{2}}\,\bar{\nu}\,\gamma^\mu(1+\gamma_5)\nu\,\{(1-\tfrac{4}{3}\sin^2\theta_W)\bar{p}\,\gamma_\mu p$$

$$+ (\tfrac{2}{3}\sin^2\theta_W - \tfrac{1}{2})\,\bar{n}\,\gamma_\mu n - \tfrac{1}{2}\bar{n}\,\gamma_\mu\gamma_5 n$$

$$+ (\tfrac{2}{3}\sin^2\theta_W - \tfrac{1}{2})\,\bar{\lambda}\,\gamma_\mu\lambda - \tfrac{1}{2}\bar{\lambda}\,\gamma_\mu\gamma_5\lambda\}.$$

To compute the cross-section ratios, we use a very naive quark-parton model, sufficient in a first approximation. We will consider the proton as formed by the quarks p, p, n and the neutron by the quarks n, n, p **.

Such an approximation leads to the value $\frac{1}{3}$ of the ratio

$$R_0 = \frac{\sigma^{CC}_{\bar{\nu}\text{-nucleon}}}{\sigma^{CC}_{\nu\text{-nucleon}}}.$$

* In the limit in which the angles β and β' are equal.

** We neglect also the difference between the numbers of protons and neutrons in the liquid (CF_3Br) of the bubble chamber Gargamelle.

The measured value is [4] 0.38 ± 0.02, indicating that the contribution of a $q\bar{q}$ sea is small. Moreover we can easily verify that the following results are only very little modified in the presence of such a sea.

The ratios R and $\bar{R}$ of the neutral current cross sections to the charged current cross sections, relative to the neutrino and the antineutrino, are given in terms of the angle θ_W by:

$$R = \frac{\sigma^{NC}_{\nu\text{-nucleon}}}{\sigma^{CC}_{\nu\text{-nucleon}}} = \tfrac{20}{27}\sin^4\theta_W - \tfrac{11}{9}\sin^2\theta_W + \tfrac{7}{12},$$

$$\bar{R} = \frac{\sigma^{NC}_{\bar{\nu}\text{-nucleon}}}{\sigma^{CC}_{\bar{\nu}\text{-nucleon}}} = \tfrac{20}{9}\sin^4\theta_W - 3\sin^2\theta_W + \tfrac{5}{4}.$$

For the value $\sin^2\theta_W = \frac{3}{8}$ *, these ratios are equal to those calculated in the Weinberg model. Moreover, all scattering cross sections of ν or $\bar{\nu}$ on p or n are equal in both models. The recent CERN experiments [5] gave the results

$$R = 0.23 \pm 0.03, \quad \bar{R} = 0.46 \pm 0.09.$$

Then we find for the neutrino

$$0.33 < \sin^2\theta_W < 0.42,$$

for the antineutrino

$$0.30 < \sin^2\theta_W < 0.43,$$

or

$$0.92 < \sin^2\theta_W.$$

The measured value of R and $\bar{R}$ are thus compatible, for a value of $\sin^2\theta_W$ verifying

$$0.33 < \sin^2\theta_W < 0.42.$$

Note that $\sin^2\theta_W = \frac{3}{8}$ is just in the middle of this interval. Then the model we propose and the Weinberg model cannot be decided upon through the neutrino-hadron experiments. We can take into account a sea of quark-antiquark pairs: the calculation becomes more model-dependent but the modifications of the bounds of this interval are only slight.

Now, we show, on a same graph, the variations of R and $\bar{R}$ in terms of θ_W, for the proposed model and the Weinberg model (see fig. 3).

We recall the results obtained for the purely leptonic weak transitions

* $\sin^2\theta_W = \frac{3}{8}$ is the value for which the coupling of Z to the quark p is purely axial in the Weinberg model.

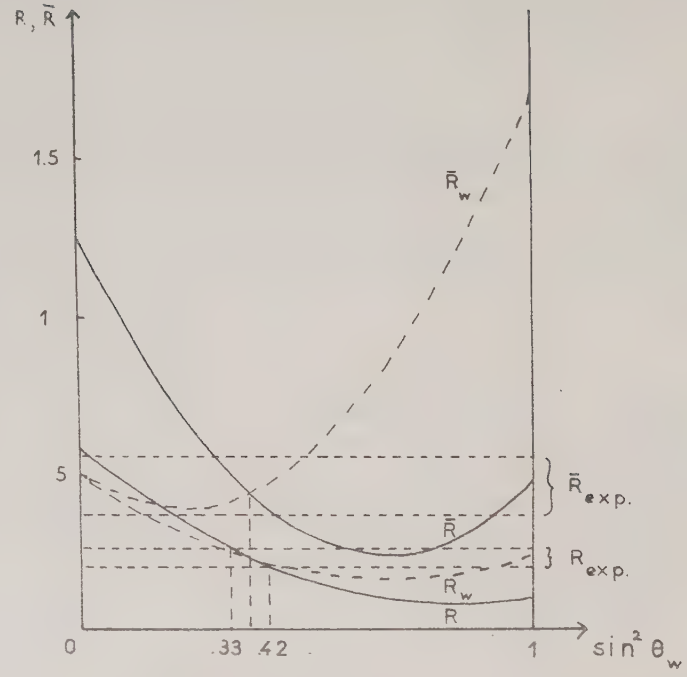

Fig. 3. Comparison of the experimental data on the ratios $R = (\mathrm{NC/CC})_{\nu}$ and $\bar{R} = (\mathrm{NC/CC})_{\bar{\nu}}$ with the predictions of the proposed model and of the Weinberg model (W).

$0.16 < \sin^2\theta_W < 0.84$ for the ν_μe diffusion ,

$0.17 < \sin^2\theta_W < 0.70$ for the $\bar{\nu}_e$e diffusion .

The proposed model allows us to interpret all the experimental results on the weak interactions, with $\sin^2\theta_W$ of order 0.4. The experimental situation is thus at least as favorable as for the Weinberg model, where it is more difficult to find an angle θ_W suitable at the same time for the ν_μe ($0.1 < \sin^2\theta_W < 0.6$), $\bar{\nu}_e$e ($\sin^2\theta_W < 0.3$) and ν_μ or $\bar{\nu}_\mu$ hadron scatterings ($\sin^2\theta_W$ from 0.3 to 0.5).

Finally, in all models involving neutral currents, the exchange of a Z between an electron and an atomic nucleus, equivalent to a four fermion interaction, leads to an electron-nucleus potential. If we neglect the terms involving the nuclear spin, the parity violating part of this potential comes from the product of the neutral leptonic axial current and hadronic vector current. In the non-relativistic approximation, it has the following expression in the Weinberg model [15, 16]:

$$V = -\frac{G}{4\sqrt{2}m_e}[\boldsymbol{\sigma}\cdot\boldsymbol{p}\,\delta^3(r) + \delta^3(r)\,\boldsymbol{\sigma}\cdot\boldsymbol{p}][(4\sin^2\theta_W - 1)Z + N] .$$

Bouchiat and Bouchiat [16] have shown that one might hope to detect such a potential by parity-violation effects in some heavy-atom transitions. On the contrary, in the proposed model, the couplings of Z with the nucleons are parity-violating, but the coupling with the electron conserves parity, and V vanishes in the above approximations.

6. Conclusion

We have presented a unified theory of weak and electromagnetic interactions, depending on the angle θ_W. The initial theory, in which two neutrinos are associated to every charged lepton, conserves parity. The latter is spontaneously broken by the Higgs mechanism, together with gauge-invariance, which gives a theory where the electromagnetism conserves parity naturally, whereas the charged currents coupling the charged leptons to the massless neutrinos violate it in a maximal way.

The couplings of Z to the charged leptons are also parity-conserving, thus we find that the ν_μe and $\bar{\nu}_\mu$e cross sections are equal; the present results give $0.16 < \sin^2\theta_W < 0.84$; and the $\bar{\nu}_e$e experiment gives $0.17 < \sin^2\theta_W < 0.70$.

The model can be extended to hadrons, by introducing three charmed quarks p′, n′ and λ′ in addition to the three usual quarks. The ν_μ and $\bar{\nu}_\mu$ hadron experiments made at CERN are then compatible, and we find that $\sin^2\theta_W$ is of order 0.4.

At the price of the introduction of the heavy neutrinos ν'_e and ν'_μ we have a model, compatible with experiment, which gives the parity conservation by electromagnetism naturally, and its (spontaneous) breaking by the usual weak interactions.

I wish to thank C. Bouchiat, J. Iliopoulos and Ph. Meyer for many helpful discussions.

Appendix

Using the invariance of the theory under the Q transformation ($\varphi' \to e^{i\alpha}\varphi'$, $\varphi'' \to e^{i\alpha}\varphi''$) and under parity, which permutes φ' and φ'' we write the general expression of the potential coupling the Higgs bosons:

$$V(\varphi', \varphi'') = a(\varphi'^{+}\varphi' + \varphi''^{+}\varphi'') + b[(\varphi'^{+}\varphi')^2 + (\varphi''^{+}\varphi'')^2]$$

$$+ c\,\varphi'^{+}\varphi'\varphi''^{+}\varphi'' + d\,\varphi'^{+}\varphi''\varphi''^{+}\varphi' .$$

In order to know when the theory is spontaneously broken, we search for the minimum for $V(\varphi', \varphi'')$. Let us write $\varphi'^{+}\varphi' = v'^2$, $\varphi''^{+}\varphi'' = v''^2$; at first v'^2 and v''^2 are kept fixed, and we minimize $d\,\varphi'^{+}\varphi''\varphi''^{+}\varphi'$; its minimum is zero if $d > 0$, or $d\,v'^2\,v''^2$ if $d < 0$ (with φ' and φ'' proportional).

We must now minimize:

$$V(\varphi', \varphi'') = a(v'^2 + v''^2) + b(v'^2 + v''^2)^2 + [c - 2b - \sup(-d, 0)]v'^2 v''^2 .$$

If the conditions

$$b > 0\,, \quad a < 0\,, \quad c - 2b > \sup(-d, 0)\,,$$

are satisfied, the minimum is obtained for

$$v'^2 + v''^2 \neq 0\,, \quad v'^2\, v''^2 = 0\,.$$

One, and only one, of the two quantities v'^2 and v''^2 is zero. We have a spontaneous breaking of both gauge-invariance and parity. We will choose

$$|\langle\varphi'\rangle| = v' \neq 0\,, \quad |\langle\varphi''\rangle| = v'' = 0\,,$$

in order to have a left polarization for the usual neutrino.

References

[1] S. Weinberg, Phys. Rev. Letters 19 (1967) 1264.
[2] H. Georgi and S.L. Glashow, Phys. Rev. Letters 28 (1972) 1494.
[3] B.W. Lee, Phys. Rev. D6 (1972) 1188;
J. Prentki and B. Zumino, Nucl. Phys. B47 (1972) 99.
[4] P. Musset, Report to the 2nd Int. Conf. on elementary particles at Aix en Provence, 1973.
[5] F.J. Hasert et al., Phys. Letters 46B (1973) 138.
[6] M.A.B. Bég and A. Zee, Phys. Rev. Letters 30 (1973) 675.
[7] F.J. Hasert et al., Phys. Letters 46B (1973) 121.
[8] H.S. Gurr, F. Reines and H.W. Sobel, Trieste Conf., 1973; Phys. Rev. Letters 28 (1973) 1406.
[9] B. Lautrup, A. Peterman and E. de Rafael, Phys. Reports 3 (1972) 193.
[10] J. Bailey et al., Phys. Letters 28B (1968) 287.
[11] S.J. Brodsky and J.D. Sullivan, Phys. Rev. 156 (1967) 1644;
T. Burnett and M.J. Levine, Phys. Letters 24B (1967) 467.
[12] J. Brodsky and E. de Rafael, Phys. Rev. 168 (1968) 1620.
[13] K. Fujikawa, B.W. Lee and A.I. Sanda, Phys. Rev. D6 (1972) 2923;
J.R. Primack and H.R. Quinn, Phys. Rev. D6 (1972) 3171.
[14] S.L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. D2 (1970) 1285.
[15] F. Curtis-Michel, Phys. Rev. 138B (1965) 408.
[16] M.A. Bouchiat and C. Bouchiat, Phys. Letters 48B (1974) 111.

INTRODUCTION TO WEAK NEUTRAL CURRENTS

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Abstract : We present the question of weak neutral currents, in connection with the unified gauge theories of weak and electromagnetic interactions.

Résumé : Nous exposons la question des courants neutres faibles, en liaison avec les théories de jauge unifiées des interactions faibles et électromagnétiques.

I. CURRENTS IN A PHENOMENOLOGICAL THEORY OF WEAK INTERACTIONS.

Weak interaction phenomena are usually described in terms of currents, but the distinction between the so-called "charged currents" and "neutral currents" is not always well-defined. For example, the effective Lagrangian density responsible for the μ-decay :

$$(1) \qquad \mathcal{L} = \frac{G}{\sqrt{2}} \left[\bar{\mu}\, \gamma^{\mu}(1+\gamma_5)\nu_{\mu}\, \bar{\nu}_e\, \gamma_{\mu}(1+\gamma_5)\, e \quad + \text{h. c.} \right]$$

can be re-written with a Fierz transformation :

$$(2) \qquad \mathcal{L} = \frac{G}{\sqrt{2}} \left[\bar{\mu}\, \gamma^{\mu}(1+\gamma_5)\, e\ \bar{\nu}_e\, \gamma_{\mu}(1+\gamma_5)\, \nu_{\mu} \quad + \text{h. c.} \right]$$

In expression (1), appears the product of two charged currents, and in (2), the product of two neutral currents. For the moment, it is arbitrary to say that the μ-decay is due to charged, or to neutral, currents.

Consider now all the classical weak interaction phenomena. They are described by an effective Lagrangian density involving the product of two charged currents :

$$(3) \qquad \mathcal{L} = \frac{G}{\sqrt{2}}\ J^{\mu\,\dagger} J_{\mu}$$

J_{μ} is the weak charged current, which has a leptonic part :

$$(4) \qquad l_{\mu} = \bar{\mu}\, \gamma_{\mu}(1+\gamma_5)\, \nu_{\mu} \ + \ \bar{e}\, \gamma_{\mu}(1+\gamma_5)\, \nu_e$$

and a hadronic part h_{μ}. The last one has no simple expression in terms of the field of known hadrons ; but its symmetry properties are well described in a simple quark model (this does not mean of course that the quark fields

represent real elementary particles) :

$$(5) \qquad h_\mu = \cos\theta_c \; \bar{n} \gamma_\mu (1+\gamma_5) p + \sin\theta_c \; \bar{\Lambda} \gamma_\mu (1+\gamma_5) p$$

where θ_c is the Cabibbo angle.

Note that it is not possible to re-write (3), as we did for (1), as a simple product of two neutral currents.

One may ask whether (3) is the whole effective Lagrangian density responsible for all the weak processes. For example, if an interaction $\sim G \; \bar{e} \gamma_\mu e \; \bar{e} \gamma^\mu e$ existed, it would have remained up to now undetected.

New weak interaction processes seem now to have been observed in the CERN and the NAL neutrino-hadron experiments[1]. They are interpreted as ν_μ + hadron → ν_μ + hadron scatterings, and a new term must be added to the effective Lagrangian density. This term is written as the product of two neutral currents, one involving the neutrino and the other the hadrons.

Another way to describe the classical weak interactions is to introduce a charged vector boson W coupled to the current J^μ

$$(6) \qquad \mathcal{L} = g \left[W_\mu^- J^\mu + h.c. \right]$$

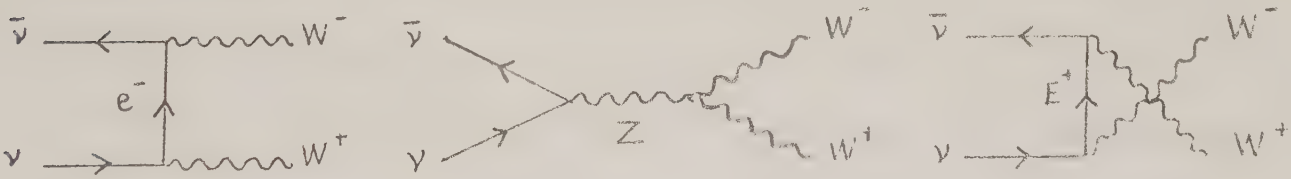
Provided the W is sufficiently heavy relatively to the energies involved, the one-W-exchange is equivalent to the local interaction described by (3).

For the ν_μ + hadron → ν_μ + hadron scattering, one may also introduce a neutral vector boson Z coupled both to a neutrino current and a hadronic current. Once the vector bosons mediating the weak interactions have been introduced, the distinction between charged and neutral currents is physical, and no more arbitrary as it was at first for the μ-decay.

II. GAUGE THEORIES AND NEUTRAL CURRENTS.

A. Gauge Theories.

It is well-known that the theory of weak interactions described by (6) is not renormalisable, and that this is connected with cross sections growing too fast with energy. In order to improve their high energy behaviour cancellations are necessary, which are obtained by adding new diagrams and therefore new particles. For example, in the $\nu + \bar{\nu} \rightarrow W^+ + W^-$ process, a cancellation must occur between diagram a), the only one of the classical theory, and new diagrams of type b) or c), where new particles are exchanged in the s- or u-channels :



a) b) c)

Z is a neutral vector boson, coupled to neutral currents. E^+ is a heavy lepton, which has the same leptonic number as the electron e^-. Thus, renormalisable theories of weak interactions must involve neutral currents and/or heavy leptons. Now such theories exist that unify both the weak and the electromagnetic interactions : they are the spontaneously-broken gauge theories.

The first theory constructed was the Weinberg-Salam model[2] which used the first possibility : no new lepton, but a neutral vector boson Z coupled to neutral currents. Such a theory predicted ν_μ + hadron → ν_μ + hadron scattering, unknown at that time ; then people were also interested in other models where such processes did not appear : in the Georgi-Glashow

model[3] there is no neutral vector boson, but four new leptons ; in the Lee Prentki-Zumino[4] models, there is a neutral vector boson Z as in the Weinberg model, but not coupled to the neutrino ; two new leptons are necessary.

Because of the results of the recent ν_μ-hadron experiments, we shall now be interested only in theories in which a neutral vector boson is coupled to a neutral current involving the neutrino, as in the Weinberg model.

B. The Weinberg Model.

Let us recall at first how one introduces the interaction in a gauge theory. We take for example electrodynamics ; the gauge group is U(1), which has a single generator Q ; there is one coupling constant, the positron charge e, and eQ is the charge operator. The interaction is defined by the minimal substitution, i.e. the replacement in the free field Lagrangian :

$$i\partial^\mu \rightarrow i\partial^\mu - eQA^\mu \tag{7}$$

A^μ is the gauge field, which represents a massless vector boson, the photon γ.

In the Weinberg model, the gauge group is $SU(2) \times U(1)$, and has four generators, $\vec{T}$ and F. There are now two coupling constants g and g' corresponding to $SU(2)$ and $U(1)$, and one writes usually :

$$\operatorname{tg}\theta_w = \frac{g'}{g} \tag{8}$$

The interaction is defined by the substitution :

$$i\partial^\mu \rightarrow i\partial^\mu - g\vec{T}\vec{W}^\mu + \frac{g'}{g}FB^\mu \tag{9}$$

where $\vec{W}^\mu$ and B^μ are four gauge fields, coupled respectively to the generators $\vec{T}$ and F of the gauge group.

$\vec{W}^\mu$ and B^μ represent four massless vector bosons, whereas only one can be massless in a realistic theory, the photon γ. Gauge invariance must then be broken. This is done "spontaneously" by the Higgs mechanism : the model contains a complex SU(2) doublet of spin-zero fields ("Higgs scalars") self-coupled in such a way that it takes a non-zero vacuum mean value. This breaks gauge invariance, and defines in SU(2) a favoured direction, the axis 3.

The combination

$$A^\mu = \sin\theta_w \, W_3^\mu + \cos\theta_w \, B^\mu \tag{10}$$

remains massless, and is identified with the photon field ; the model conserves the usual QED gauge invariance unbroken. The orthogonal combination

$$Z^\mu = \cos\theta_w \, W_3^\mu - \sin\theta_w \, B^\mu \tag{11}$$

is now the field of a massive neutral vector boson, the Z, coupled to weak neutral currents. The fields $W_\pm^\mu$ represent the ordinary charged bosons of weak interactions, which also become massive

We arrive now at the lepton sector of the model. No unknown lepton field is introduced. We denote by the index L (R) the $\frac{1+\gamma_5}{2}$ ($\frac{1-\gamma_5}{2}$) part of the fields. $\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$ is an SU(2) doublet, and e_R an SU(2) singlet; they have respectively the U(1) quantum number $F = 1$ and $F = 0$.

The charged W pair is coupled to the leptonic charged current (1), as in the classical weak interaction lagrangian density (6). The couplings of the neutral vector bosons Z and γ are given by :

$$(12)\qquad \mathcal{L} = \sqrt{g^2+g'^2}\,\sin\theta_w\cos\theta_w\,A^\mu\,\bar{e}\,\gamma_\mu e - \frac{1}{2}\sqrt{g^2+g'^2}\,Z^\mu$$
$$\left[\bar{\nu}_L\gamma_\mu\nu_L + (2\sin^2\theta_w - 1)\,\bar{e}_L\gamma_\mu e_L + 2\sin^2\theta_w\,\bar{e}_R\gamma_\mu e_R\right]$$

Note that γ is coupled to a purely vector electromagnetic current ; this is in fact a consequence of the choice of $F = 2$ for the field e_R . The electron charge is $(-e)$, where

$$(13)\qquad e = \sqrt{g^2+g'^2}\,\sin\theta_w\cos\theta_w = g\,\sin\theta_w$$

is the elementary charge. The neutral massive vector boson Z is coupled to a V-A neutrino current, and to an electron current which has both vector and axial parts.

The study of the μ-decay gives $\frac{G}{\sqrt{2}} = g^2/8M_W^2$. g is lower bounded by e , and then M_W is also lower bounded. More precisely, we have :

$$(14)\qquad M_W = \frac{37\ \mathrm{GeV}/c^2}{\sin\theta_w} \qquad M_Z = \frac{37\ \mathrm{GeV}/c^2}{\sin\theta_w\cos\theta_w}$$

C. A Model With A Spontaneous Parity-Breaking.

We present also briefly another $SU(2)\times U(1)$ model, in which parity is spontaneously broken, together with gauge invariance[5]. Beyond the known leptons e, ν_e, μ , ν_μ, two heavy leptons ν'_e and ν'_μ are introduced. The original theory is gauge- and parity-invariant, and the leptons belong to a doublet D and a singlet S of the $SU(2)$ group, which have respectively the $U(1)$ quantum numbers $F = 1$ and $F = 0$. Both gauge- and parity-invariances are spontaneously broken by the Higgs mechanism : φ' and φ'' are two Higgs $SU(2)$ doublets, exchanged by parity, and only one takes a non-zero vacuum mean value;

after the breaking, D and S are re-written in terms of the physical fields e, ν , ν' :

$$D = \begin{bmatrix} \nu_L \sin\beta_e + \nu'_L \cos\beta_e + \nu'_R \\ e \end{bmatrix}$$

(15)

$$S = \nu_L \cos\beta_e - \nu'_L \sin\beta_e + \nu_R$$

Among the different currents obtained, one has both a purely vector electromagnetic current, and a V-A charged current coupling the electron to the neutrino. A zeroth order mass relation $m_e = m_{\nu'_e} \cos\beta_e$ fixes, for the angle β_e a value near $\pi/2$, and similarly for β_μ ; an approximate e-μ universality is then ensured, since $\sin\beta_e \simeq \sin\beta_\mu \simeq 1$.

Everything concerning the gauge boson sector is like in the Weinberg model. Formula (12) giving the couplings of γ and Z to the electron and the neutrino is now replaced (with the approximation $\sin\beta \simeq 1$) by :

$$\mathcal{L} = \sqrt{g^2 + g'^2}\, \sin\theta_w \cos\theta_w\, A^\mu \bar{e} \gamma_\mu e - \frac{1}{2}\sqrt{g^2 + g'^2}\, Z^\mu \left[\bar{\nu}_L \gamma_\mu \nu_L + (2\sin^2\theta_w - 1)\, \bar{e}\gamma_\mu e \right] \qquad (16)$$

The only difference between (12) and (16) is the coupling of Z with the right-handed electron current. In (16) Z is coupled to a purely vector electron current, whereas in the Weinberg model, the coupling of Z to the electron current is parity-violating.

III. NEUTRAL CURRENTS IN NEUTRINO-ELECTRON SCATTERINGS.

A. $\nu_\mu e$ And $\bar{\nu}_\mu e$ Scatterings.

These processes are forbidden at order G in the classical V-A theory, as well as in renormalisable theories without neutral currents involving the

neutrino, such as the Georgi-Glashow and the Lee-Prentki-Zumino models ; in these models, the second order weak interaction contributions can be evaluated, and are found to be $\sim G\alpha$. On the contrary, such processes are allowed at order G in theories in whicha neutral vector boson is coupled both to electron and neutrino currents, as the two SU(2) × U(1) models presented in Section II. For these we define the coefficients g_V and g_A by re-writing the Z-couplings given in (12) or (16), in the following form :

$$(17)\qquad \mathcal{L} = -\frac{1}{2}\sqrt{g^2+g'^2}\, Z^\mu \left\{ \bar{\nu}_L \gamma_\mu \nu_L + \bar{e}\gamma_\mu (g_V + g_A\gamma_5) e \; + \left[e \leftrightarrow \mu \text{ terms} \right] \right\}$$

The $\nu_\mu e$ and $\bar{\nu}_\mu e$ amplitudes can be described at order G, in the local limit by the effective Lagrangian density :

$$(18)\qquad \mathcal{L}_{eff} = \frac{G}{\sqrt{2}}\; \bar{\nu}_\mu \gamma^\mu (1+\gamma_5)\nu_\mu \;\; \bar{e}\gamma_\mu (g_V + g_A\gamma_5) e$$

Let ρ be the ratio of the recoil electron energy to the incoming neutrino energy E ; the differential cross section is[6] :

$$(19)\qquad \frac{d\sigma^{\left\{\begin{smallmatrix}\nu_\mu e\\ \bar{\nu}_\mu e\end{smallmatrix}\right.}}{d\rho} = \frac{G^2 m E}{2\pi}\left[(g_V \pm g_A)^2 + (1-\rho)^2 (g_V \mp g_A)^2 + \frac{m}{E}\rho\,(g_A^2 - g_V^2)\right]$$

Neglecting m/E and integrating over ρ, we find :

$$(20)\qquad \sigma = \frac{G^2 m E}{2\pi}\left[(g_V \pm g_A)^2 + \frac{1}{3}(g_V \mp g_A)^2 \right]$$

As unit, we take the $\bar{\nu}_e e$ cross section in the V-A theory :

$$(21)\qquad \sigma_0 = \sigma^{V-A}_{\bar{\nu}_e e} = \frac{G^2 m E}{2\pi} \times \frac{4}{3} \simeq .56\; 10^{-41}\, cm^2\; E\,(GeV)$$

In the CERN Gargamelle experiment one $\bar{\nu}_\mu e$ event had already been observed[7], and we learnt in this meeting that a second $\bar{\nu}_\mu e$ event has been found recently. The following bounds have been given[1] :

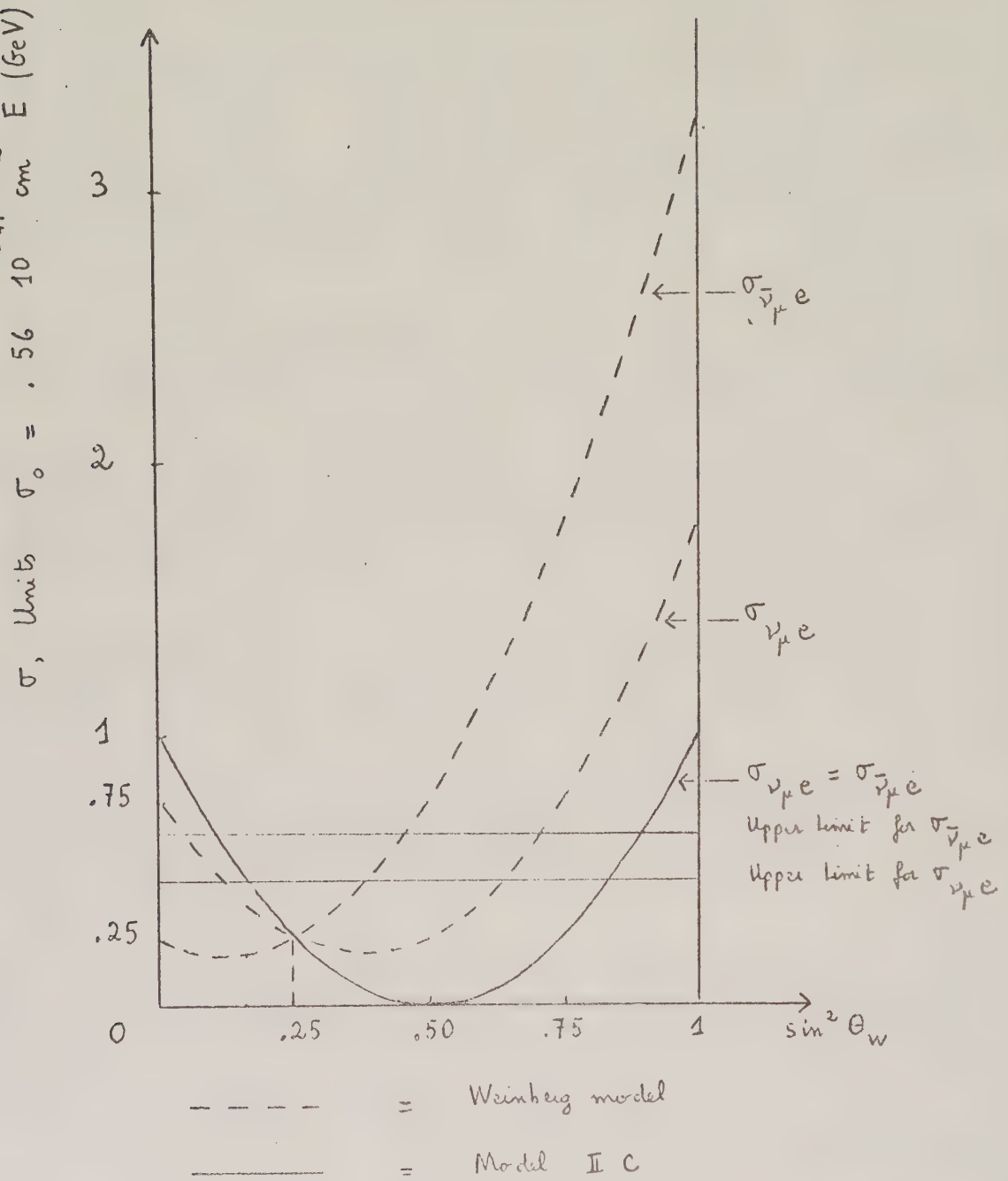
$$\sigma_{\nu_\mu e} < .26 \quad 10^{-41} \text{ cm}^2 \quad E \text{ (GeV)}$$

$$\sigma_{\bar{\nu}_\mu e} < .35 \quad 10^{-41} \text{ cm}^2 \quad E \text{ (GeV)} \qquad (22)$$

Now we compare these experimental results with the theoretical predictions of the Weinberg model, and of the model described in Section II.C.

	Weinberg Model	Model II.C.
g_V	$2 \sin^2\theta_W - \frac{1}{2}$	$2 \sin^2\theta_W - 1$
g_A	$-\frac{1}{2}$	0
$\sigma_{\nu_\mu e}/\sigma_o$	$4 \sin^4\theta_W - 3 \sin^2\theta_W + \frac{3}{4}$	$(2 \sin^2\theta_W - 1)^2$
$\sigma_{\bar{\nu}_\mu e}/\sigma_o$	$4 \sin^4\theta_W - \sin^2\theta_W + \frac{1}{4}$	$(2 \sin^2\theta_W - 1)^2$
$\sin^2\theta_W$	.11 to .46	.16 to .84

More precisely, a maximum likelihood calculation gives for the Weinberg model $.05 < \sin^2\theta_W < .5$, with 90% confidence[1]. Notice also that in model II.C., the region $\sin^2\theta_W \simeq .5$ is probably to be excluded, since it gives $\sigma_{\nu_\mu e} = \sigma_{\bar{\nu}_\mu e} \simeq 0$.



For $\sin^2\theta_W = \frac{1}{4}$, the $\nu_\mu e$ and $\bar{\nu}_\mu e$ cross sections in both models are equal; and their common value is $\sigma_o/4$: This can be understood very simply: for each value of θ_W, the sum $(g_V + g_A)$ is the same in both models as it gives the coupling of Z to the left-handed electron current ; $\sin^2\theta_W = \frac{1}{4}$ is the value for which Z is coupled to a purely axial electron current in

the Weinberg model, and then :

$$(g_A)_{\text{Weinberg}} = (g_V)_{\text{model II.C.}}$$

whereas

$$(g_V)_{\text{Weinberg}} = (g_A)_{\text{model II.C.}} = 0$$

From this follows the equality of cross sections.

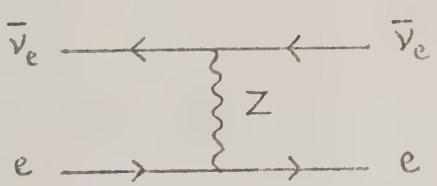
The essential character of model II.C. is to predict equal ν_μ e and $\bar{\nu}_\mu$ e cross sections, whatever θ_W.

B. $\bar{\nu}_e$ e Scattering.

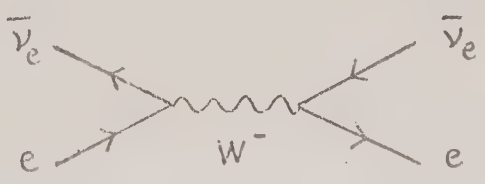
This experiment has been made at the Savannah River Reactor[8]. The problem is that the $\bar{\nu}_e$ flux cannot be directly measured, but only calculated; and the result is very dependent on the spectrum. The value reported at the Trieste Conference is

$$\frac{\sigma_{\bar{\nu}_e e}}{\sigma^{V-A}_{\bar{\nu}_e e}} = 1.1 \pm .8 \qquad (23)$$

Either Z or W can be exchanged in this process :



a)



b)

and at order G it can be described by the effective Lagrangian density

(24) $$\mathcal{L} = \frac{G}{\sqrt{2}} \bar{\nu}_e \gamma^\mu (1+\gamma_5) \nu_e \quad \bar{e} \gamma_\mu (g_V + g_A \gamma_5) e$$

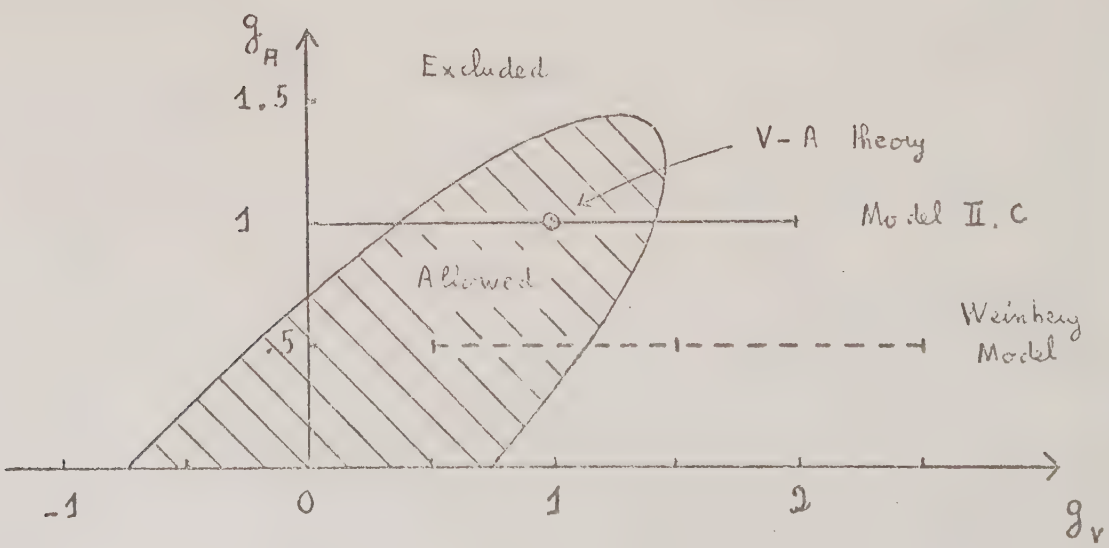
where g_V and g_A are obtained by adding 1 (because of diagram b) to those of Section III.A.

The limits given on g_V and g_A

(25) $$\left(\frac{g_V + g_A}{2}\right)^2 + 14 \left(\frac{g_V - g_A}{2}\right)^2 < 2$$

are compatible with the V-A theory. We compare them with the predictions of models involving neutral currents :

	Weinberg Model	Model II.C.
g_V	$2 \sin^2\theta_W + \frac{1}{2}$	$2 \sin^2\theta_W$
g_A	$\frac{1}{2}$	1
$\sin^2\theta_W$	$< .3$	.17 to .70

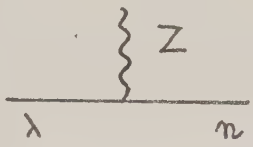


Both the Weinberg model and model II.C. are then compatible with the present experimental data on purely leptonic transitions.

IV. HADRONIC NEUTRAL CURRENTS.

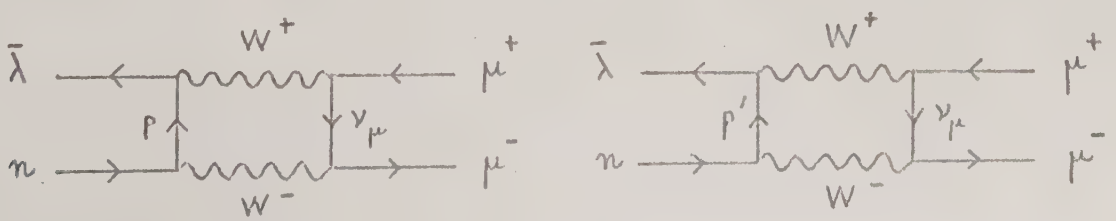
A. The Problem of Strangeness-Changing Neutral Currents.

Forgetting about strangeness, one extends the Weinberg model to hadrons, taking $\begin{pmatrix} p_L \\ n_L \end{pmatrix}$ as a SU(2) doublet, whereas p_R and n_R are SU(2) singlets. If we introduce strangeness in the usual way, with the Cabibbo rotation $n \to n(\theta_c) = n \cos\theta_c + \lambda \sin\theta_c$, we obtain the following coupling :



Such a coupling of Z to a strangeness changing neutral current would lead to values of the $K_L - K_S$ mass splitting, or the $K_L \to \mu\bar{\mu}$ and $K_L \to \pi\nu\bar{\nu}$ decay amplitudes, much larger than those measured experimentally. A cancellation mechanism must be found. In the Glashow-Iliopoulos-Maiani mechanism[9], a fourth quark p' is introduced, and $\begin{pmatrix} p_L \\ n_L(\theta_c) \end{pmatrix}$ and $\begin{pmatrix} p'_L \\ \lambda_L(\theta_c) \end{pmatrix}$ are two SU(2) doublets. Z is no longer coupled to a strangeness changing neutral current, ensuring that the $K \to \bar{K}$, $K \to \mu\bar{\mu}$, $K \to \pi\nu\bar{\nu}$ amplitudes vanish at order G.

These amplitudes also vanish at order $G\alpha$ in the limit of exact SU(4) : for example, the two following diagrams of the $K \to \mu\bar{\mu}$ decay cancel exactly in this limit :



The order of magnitude of the SU(4) breaking must be small compared with the W mass, if we want the cancellation mechanism to work, and the amplitudes to be small enough, in agreement with the experimental measurements.

B. Interpretation of the Neutrino-Hadron Scattering Experiments.

The exchange of a neutral vector boson Z between a neutrino line and a quark line can be described, in the Weinberg model, by the effective Lagrangian density :

$$(26)\qquad \mathcal{L}_{eff} = \frac{G}{\sqrt{2}}\,\bar{\nu}_\mu \gamma^\mu (1+\gamma_5)\nu_\mu \left[\bar{p}\,\gamma_\mu\left(\frac{1}{2} - \frac{4}{3}\sin^2\theta_w + \frac{\gamma_5}{2}\right)p + \bar{n}\,\gamma_\mu\left(-\frac{1}{2} + \frac{2}{3}\sin^2\theta_w - \frac{\gamma_5}{2}\right)n\right]$$

This expression is valid in the local limit, and the charmed and strange quarks have been ignored.

To compute the cross section ratios,we use a very naive quark parton model, sufficient in a first approximation : the proton is considered as composed of the quarks $p, \bar{p}, n$ and the neutron of quarks $n, \bar{n}, p$. Such an approximation leads to the value $\frac{1}{3}$ of the ratio $R_o = \dfrac{\sigma^{CC}_{\bar{\nu}\text{-Nucleon}}}{\sigma^{CC}_{\nu\text{-Nucleon}}}$. The measured value is $.38 \pm .02$, indicating that the contribution of a $q\bar{q}$ sea is small.

Neglecting the Cabibbo angle and the difference between the numbers of protons and neutrons in the Gargamelle liquid (CF_3Br), we compute the ratios :

$$R = \frac{\sigma^{NC}_{\nu N}}{\sigma^{CC}_{\nu N}} \qquad \bar{R} = \frac{\sigma^{NC}_{\bar{\nu} N}}{\sigma^{CC}_{\bar{\nu} N}}$$

Similar calculations are made in model II.C. which is extended to hadrons introducing the six quarks p , n , λ , p' , n' , λ' in analogy with the six leptons. Formula (26) must be replaced by

$$(27) \qquad \mathcal{L}_{\mathcal{H}} = \frac{G}{\sqrt{2}} \bar{\nu}_\mu \gamma^\mu (1+\gamma_5) \nu_\mu \left[\left(1 - \frac{4}{3} \sin^2\theta_W \right) \bar{p} \gamma_\mu p + \bar{n} \gamma_\mu \left(-\frac{1}{2} + \frac{2}{3} \sin^2\theta_W - \frac{\gamma_5}{2} \right) n \right]$$

Both models are equivalent for the neutrino-n-scattering. For the neutrino-p-scattering, they are equivalent if $\sin^2\theta_W$ has the particular value $\frac{3}{8}$, for which the coupling of Z to p is purely axial in the Weinberg model, whereas it is purely vector in model II.C. The knowledge of R and $\bar{R}$, if $\sin^2\theta_W \simeq 3/8$, cannot allow to decide between the two models, even if a proton-neutron separation is made, with the propane experiment.

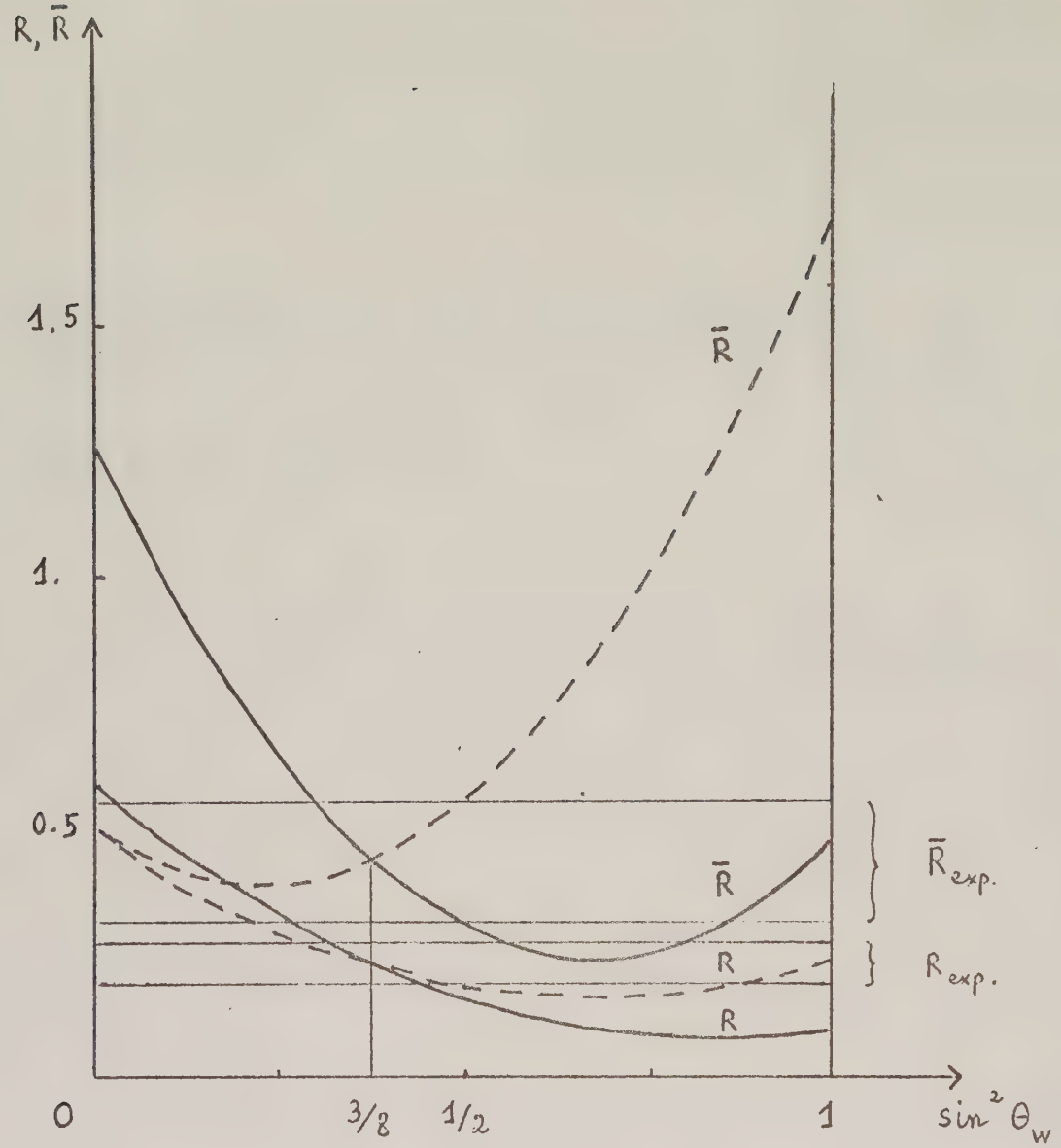
The experimental values of R and $\bar{R}$ are[1, 10]:

$$R = .23 \pm .04$$

$$\bar{R} = .43 \pm .12$$

The comparison with the theoretical predictions is given in the following table : ($S = \sin\theta_W$)

	Weinberg model	Model II.C.
R	$\frac{20}{27} S^4 - S^2 + \frac{1}{2}$	$\frac{20}{27} S^4 - \frac{11}{9} S^2 + \frac{7}{12}$
$\bar{R}$	$\frac{20}{9} S^4 - S^2 + \frac{1}{2}$	$\frac{20}{9} S^4 - 3 S^2 + \frac{5}{4}$
$\sin^2\theta_W$	.29 to .48	.32 to .44



As $\sin^2\theta_W = 3/8$ is just in the range suggested by experiment, the two models may probably not be distinguished by the neutrino-hadron experiments.

More detailed (but model-dependent) calculations have been made for the Weinberg model[11], taking into account the effect of a sea of quark-anti quark pairs, and of the experimental cut at 1 GeV. The result is not very much modified :

$$\sin^2\theta_W = .35 \pm .07$$

In conclusion, both the Weinberg model and model II.C are compatible with present experimental data. Further results will be obtained with the neutrino propane experiments, which may enable to distinguish the proton and neutron contributions.

V. NEUTRAL CURRENTS WITHOUT NEUTRINOS.

We come now to some effects of neutral currents, outside neutrino physics.

A. High-Energy Scatterings.

The exchange of a neutral vector boson Z may contribute to the $e^+ e^-$ annihilation, and even dominate the one-photon exchange if s is near M_z^2 But the order of magnitude expected for M_z in the usual gauge theories ($M_z > 75$ GeV/c^2 in the Weinberg model) compels to search for Z-effects at lower energies. Although the one-photon exchange is then dominant, one can look for the interference term between the one-photon and the Z-exchanges.

Let θ be the angle between the e^- and the observed particle, for example a μ^- or π^-. The one-photon exchange is symmetric in $\cos\theta$, but the interference term is not if Z is coupled with both an axial electron current and an axial muon (or hadronic) current ; the differential cross section is written :

$$\frac{d\sigma}{d\cos\theta} \sim 1 + \cos^2\theta + K\cos\theta$$

The coefficient K gives the asymmetry between the forward and backward hemispheres. When it is not identically zero (as in the model of

Section II.C where Z is coupled to a purely vector electron current), K is usually $\sim \frac{s}{s - M_Z^2}$, and the expected asymmetries are $\sim 1\%$ for $s = (10\ \text{GeV})^2$.

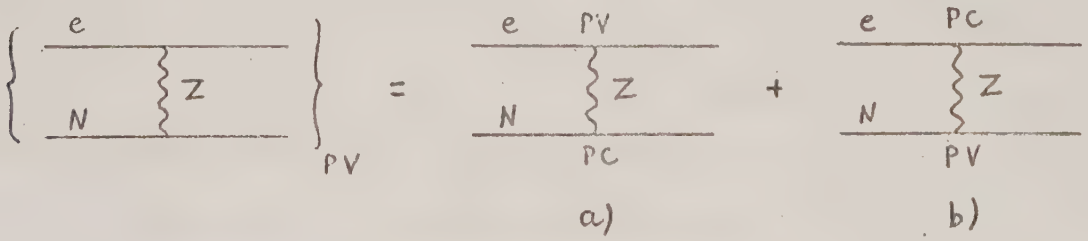
But there are other sources of asymmetries, such as the Bremstrahlung contribution, and the interference term between the one-photon exchange and the two-photon exchange ; both are $\sim \alpha$, and they give important asymmetries especially around $\theta = 0$. In order to separate the weak asymmetry, a very careful examination of the Q.E.D. terms is required ; a part of these can be eliminated if one uses the fact that e^+ and e^- are polarized. Similar discussions can also be made for the p.p -scattering[13].

In electroproduction experiments with polarized electrons or muons, asymmetry measurements may in principle allow to detect the parity-violation effects of a Z exchange, but the expected effects are small[14].

B. Parity-Violation Effects in Atomic Physics.[15]

A Z-exchange between an electron and a nucleus may lead to a very short-range parity-violating potential, which has generally two parts :

(29)



In the non-relativistic approximation term b) involves the nuclear spin, whereas term a) is proportional to a linear combination Q_W of the numbers of protons and neutrons in the nucleus and therefore is dominant in heavy atoms.

Parity-mixing occurs between atomic levels (S and P states because of the short range of the parity-violating potential). The matrix element of the P.V. potential appears to be proportional to $Z^2\ Q_W$, thus the interest of

working on heavy atoms.

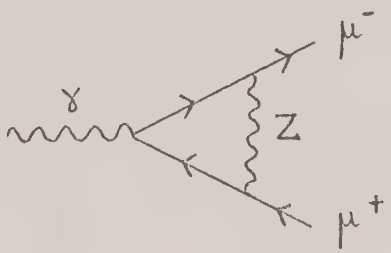
A way to investigate such parity-mixings is to look, in radiative transitions between unpolarized states, for the dependence of the transition probability on the state of circular polarisation of the emitted photon. For instance, a transition which would normally be a magnetic dipole one with an amplitude M_1 gets a small electric dipole component E_1, and there appears a circular polarisation P_c :

$$\left| P_c \right| \simeq 2 \left| \frac{E_1}{M_1} \right|$$

M.A. and C. Bouchiat[15] found that, for a suitable choice of the atomic levels the expected polarisation was of order 10^{-4}, which one may hope to detect. Of course, in a model such as that of Section II.C, the term a) in (29) is identically zero, and P_c vanishes in the above approximation.

C. Static Effects.

The graph



gives a correction to the muon anomalous magnetic moment[16]. Its contribution to the muon anomaly in usual gauge theories such as the Weinberg model is found to be $\sim \frac{G\, m_\mu^2}{8\, \pi^2 \sqrt{2}} \sim 10^{-9}$. The present experimental uncertainty is $\sim 3.\ 10^{-7}$; even if the next CERN experiment improves it by a factor 20, the weak current effect will remain negligible.

VI. CONCLUSION.

The possible existence of weak neutral currents was a very important question in elementary particle physics. People were more motivated to search them, as the gauge theories of weak and electromagnetic interactions required the introduction of heavy leptons or neutral currents. It seems now that such currents have been observed in the neutrino-hadron scattering, and this is, independently of any theoretical consideration, a fundamental experimental fact.

In order to interpret the absence of strangeness-changing neutral currents, the introduction of a new quantum number, the "charm", has been proposed ; the possible existence of "charmed" particles is also an essential field to investigate. Generally, the lepton sectors of gauge theories may be considered as more "credible" than the hadron sectors, and the most reliable tests are the purely leptonic transitions. There exist gauge theories in agreement with experimental data, and first the Weinberg model ; but at present these data are not restrictive enough to draw any conclusion ; the equality or inequality of the $\nu_\mu e$ and $\bar{\nu}_\mu e$ cross sections, and a proton/neutron separation in the neutrino-hadron scattering with the propane experiment, would be further tests of these models. Finally, one can search for the effects of neutral currents, especially the axial ones, outside neutrino physics.

NOTES AND REFERENCES

(1) This question has been reviewed by L. Kluberg and D.D. Reeder in this meeting.

(2) S. Weinberg. Phys. Rev. Letters 19, 1264, (1967)
A. Salam. Proceedings of the Eighth Nobel Symposium. J. Wiley & Sons Ed. New York, 1968.

(3) H. Georgi and S.L. Glashow. Phys. Rev. Letters 28, 1494, (1972).

(4) B.W. Lee. Phys. Rev. D6, 1188, (1972)
J. Prentki and B. Zumino. Nucl. Phys. B47, 99, (1972).

(5) P. Fayet. Orsay preprint LPTHE 74/9
See also M.A. Bég and A. Zee. Phys. Rev. Letters 30, 675, (1973).

(6) G. 't Hooft. Physics Letters 37B, 195, (1971).

(7) F.J. Hasert et al. Physics Letters 46B, 121, (1973).

(8) H.S. Gurr, F. Reines and H.W. Sobel. Trieste Conference 1973,
See also Phys. Rev. Letters 28, 1406, (1973).

(9) S.L. Glashow, J. Iliopoulos and L. Maiani. Phys. Rev. D2, 1285, (1970).

(10) F.J. Hasert et al. Preprint TC-L/Int.74.1.

(11) Ref. (15) and (16) of S. Weinberg's report at the Aix Conference 1973,
L.M. Seghal Physics Letters 48B, 133, (1974),
J.P. Vialle. Orsay thesis 1974.

(12) Ref. (96)-(101) of C. Llewellyn Smith's report CERN-Th.1710,
D.A. Dicus. Phys. Rev. D8, 890, (1973),
R. Budny and A. McDonald. Physics Letters 48B, 423,(1974).

(13) R.W. Brown, K.O. Mikaelian and M.K. Gaillard. NAL-Pub. 74/14 THY.

(14) Ref. (91)-(93) of C. Llewellyn Smith's report CERN-Th.1710
C.P. Korthals-Altes, M. Perrottet and E. de Rafael. Preprint Centre de Physique Théorique, Marseille.

(15) M.A. and C. Bouchiat. Physics Letters 48B, 111, (1974).

(16) See, for example, B. Lautrup's report at the Eighth Rencontre de Moriond, I, 1973.

As a general reference on gauge theories, see also J. Iliopoulos's paper at the Eighth Rencontre de Moriond, I, 1973.

SPONTANEOUSLY BROKEN SUPERGAUGE SYMMETRIES AND GOLDSTONE SPINORS

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We present a model with a spontaneously broken supergauge symmetry which results in the appearance of a massless Goldstone spinor. The model combines supergauge invariance with ordinary gauge invariance. After the breaking the gauge boson acquires a mass as a result of the Higgs mechanism.

Recently a class of transformations has been studied in four dimensional field theories with the property that the infinitesimal parameters are totally anticommuting spinors [1, 2]. As a result any representation must contain both bosons and fermions. Field theory models invariant under such supergauge transformations have been constructed and the renormalization of some of them has been studied in detail [3, 4]. As it was shown in ref. [3] in a supergauge invariant theory all particles in a given representation must have the same mass. This follows from the relation between four dimensional translations and the anticommutators of the supergauge generators. This property considerably restricts the applicability of such theories to particle physics unless an appropriate method of symmetry breaking is found. In ref. [3], an explicit breaking by a linear term was introduced which was sufficiently 'soft' as to preserve all renormalization properties. However, it would be very interesting to study a spontaneously broken supergauge invariant model. In this paper we present an example of such a spontaneous breaking ‡1. We show that the resulting massless Goldstone particle is a spinor as one would expect from the spinorial character of the current [3, 5].

Our example is taken from the supergauge invariant extension of quantum electrodynamics found by Wess and Zumino [6]. All unexplained notation in the present paper can be found in ref. [6]. The model combines the ordinary abelian gauge invariance with supergauge invariance. It follows that the role of the photon is taken by a whole vector multiplet V which contains a vector field V^μ ('photon'), a spinor λ and several other fields which, however, can be eliminated and do not represent physical particles. The vector multiplet V interacts with two real scalar multiplets S_1 and S_2 which can be viewed as the real and imaginary parts of a complex scalar one.

Let $C, \chi, M, N, V^\mu, \lambda$ and D be the members of the vector multiplet and A_i, B_i, ψ_i, F_i and G_i $(i = 1, 2)$ the members of the two scalar multiplets. Their transformation properties under infinitesimal supergauge transformations with a spinor parameter α are given in ref. [1] ‡2. We recall that a D member of a vector multiplet, like the F and G members of a scalar one, changes, under supergauge transformations, by a four-derivative. In ref. [6], it was shown that supergauge invariance and ordinary gauge invariance imply the invariance under a set of generalized gauge transformations where the infinitesimal c-number parameter function $\Lambda(x)$ is replaced by a whole scalar multiplet S

$$\delta V = \partial S, \qquad \delta S_1 = g S S_2, \qquad \delta S_2 = -g S S_1, \tag{1}$$

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‡1 In the appendix of ref. [3] a heuristic argument was presented suggesting that there could be no spontaneous breaking of supergauge symmetry. However, the argument made use of the positivity of the metric and therefore it did not apply to theories containing ordinary gauge symmetries, as it is the case in our example.

‡2 We shall only consider restricted supergauge transformations with constant parameters α.

∂S is a vector multiplet constructed out of S (see ref. [6] eq. (6)) and $S S_i$ is the scalar product of two scalar multiplets introduced in ref. [1]. An important observation is that the D member of V is invariant under (1).

The Lagrangian density introduced in ref. [6] has the following form

$$L = \tfrac{1}{4}\{V_a \exp(2gV) + V_b \exp(-2gV)\}_{D\,\text{component}} + \tfrac{1}{2}m(S_1 S_1 + S_2 S_2)_{F\,\text{component}} - \tfrac{1}{2}(V, V)_{F\,\text{component}} \quad (2)$$

where V_a and V_b are two vector multiplets which are bilinear in S_1 and S_2 and whose explicit form is given in ref. [6], eqs. (13) to (18). (V, V) is the scalar multiplet constructed out of V which was introduced in ref. [1] eq. (14). $-\frac{1}{2}(V, V)_{F\,\text{component}}$ is just the kinetic energy term of V which is both gauge and supergauge invariant.

Eq. (2) gives the most general gauge and supergauge invariant Lagrangian which conserves parity and has terms with dimensions less or equal to four. However, if one wants to abandon parity conservation, one can add further terms the simplest one being a term proportional to the D member of V which, as we noticed, is both gauge and supergauge invariant[‡3]. One can still add a term proportional to the G component of $(S_1 S_1 + S_2 S_2)$ but we shall restrict ourselves to the term D which is the one with the lowest dimensionality and which it will turn out to be sufficient to trigger a spontaneous symmetry breaking mechanism. No essentially new phenomena appear with the introduction of further terms.

Finally, as it was shown in ref. [6], we can choose a special gauge in which all the components of V with the exception of V^μ, λ and D, vanish. In this case (2) takes the simple form (see ref. [6] eq. 26)

$$\begin{aligned} L = &-\tfrac{1}{2}[(\partial A_1)^2 + (\partial A_2)^2 + (\partial B_1)^2 + (\partial B_2)^2 - F_1^2 - F_2^2 - G_1^2 - G_2^2 + i\bar\psi_1 \partial\!\!\!/ \psi_1 + i\bar\psi_2 \partial\!\!\!/ \psi_2] \\ &+ m(F_1 A_1 + F_2 A_2 + G_1 B_1 + G_2 B_2 - \tfrac{1}{2} i\bar\psi_1 \psi_1 - \tfrac{1}{2} i\bar\psi_2 \psi_2) - \tfrac{1}{4} V_{\mu\nu}^2 - \tfrac{1}{2}\bar\lambda \partial\!\!\!/ \lambda + \tfrac{1}{2} D^2 \\ &+ g[D(A_1 B_2 - A_2 B_1) - V_\mu(A_1 \partial^\mu A_2 - A_2 \partial^\mu A_1 + B_1 \partial^\mu B_2 - B_2 \partial^\mu B_1 - i\bar\psi_1 \gamma^\mu \psi_2) \\ &- i\bar\lambda[(A_1 + \gamma_5 B_1)\psi_2 - (A_2 + \gamma_5 B_2)\psi_1]] - \tfrac{1}{2} g^2 V_\mu^2 (A_1^2 + A_2^2 + B_1^2 + B_2^2) + \xi D, \end{aligned} \quad (3)$$

where we have added the parity-breaking term ξD. From (3) we can eliminate the auxiliary fields F_1, F_2, G_1, G_2 and D and we find

$$\begin{aligned} L = &-\tfrac{1}{2}[(\partial A_1)^2 + (\partial A_2)^2 + (\partial B_1)^2 + (\partial B_2)^2 + i\bar\psi_1 \partial\!\!\!/ \psi_1 + i\bar\psi_2 \partial\!\!\!/ \psi_2] \\ &- \tfrac{1}{4} V_{\mu\nu}^2 - \tfrac{1}{2} i\bar\lambda \partial\!\!\!/ \lambda - \tfrac{1}{2} m^2 (A_1^2 + A_2^2 + B_1^2 + B_2^2) - \xi g(A_1 B_2 - A_2 B_1) - \tfrac{1}{2} im(\bar\psi_1 \psi_1 + \bar\psi_2 \psi_2) \\ &- g[V_\mu(A_1 \partial^\mu A_2 - A_2 \partial^\mu A_1 + B_1 \partial^\mu B_2 - B_2 \partial^\mu B_1 - i\bar\psi_1 \gamma^\mu \psi_2) + i\bar\lambda[(A_1 + \gamma_5 B_1)\psi_2 - (A_2 + \gamma_5 B_2)\psi_1]] \\ &- \tfrac{1}{2} g^2 [V_\mu^2 (A_1^2 + A_2^2 + B_1^2 + B_2^2) + (A_1 B_2 - A_2 B_1)^2]. \end{aligned} \quad (4)$$

The important difference between our eq. (4) and eq. (1) of ref. [6] is that now we have obtained non diagonal mass terms among the scalar fields. By diagonalizing we obtain the fields $\tilde A_1, \tilde A_2, \tilde B_1$ and $\tilde B_2$ given by

$$\tilde A_1 = (A_1 + B_2)/\sqrt{2}, \qquad \tilde A_2 = (A_2 + B_1)/\sqrt{2}, \qquad \tilde B_1 = (B_1 - A_2)/\sqrt{2}, \qquad \tilde B_2 = (B_2 - A_1)/\sqrt{2} \quad (5)$$

with mass terms

$$-\tfrac{1}{2}(m^2 + \xi g)(\tilde A_1^2 + \tilde B_1^2) - \tfrac{1}{2}(m^2 - \xi g)(\tilde A_2^2 + \tilde B_2^2). \quad (6)$$

We also write the quartic self coupling of the scalars as

‡3 We use the word 'invariant' although all these terms are really invariant up to four-derivatives. What we mean is that the corresponding action integrals are invariant.

$$-\tfrac{1}{8}g^2(\widetilde{A}_1^2-\widetilde{A}_2^2+\widetilde{B}_1^2-\widetilde{B}_2^2)^2. \tag{7}$$

We notice that already we have obtained a spontaneous breaking of the supergauge invariance since the masses of the scalar and spinor members of the scalar multiplets are no more the same. One can easily verify that the corresponding Goldstone particle is the massless spinor λ.

At this stage we can distinguish two cases. Let us take, without loss of generality, $\xi g > 0$. From (6) and (7) we see that if $m^2 - \xi g > 0$, the origin is an absolute minimum of the potential of the scalar fields and consequently the theory can be correctly quantized using this language. Ordinary gauge invariance is unbroken and V_μ remains massless. Its couplings to the other fields are given by

$$+gV_\mu(\widetilde{A}_1\partial^\mu\widetilde{B}_1-\widetilde{B}_1\partial^\mu\widetilde{A}_1+\widetilde{B}_2\partial^\mu\widetilde{A}_2-\widetilde{A}_2\partial^\mu\widetilde{B}_2+\mathrm{i}\bar{\psi}_1\gamma^\mu\psi_2)-\tfrac{1}{2}g^2V_\mu^2(\widetilde{A}_1^2+\widetilde{A}_2^2+\widetilde{B}_1^2+\widetilde{B}_2^2). \tag{8}$$

If however $m^2 - \xi g < 0$, then a spontaneous symmetry breaking of ordinary gauge symmetry is induced.

From this point the analysis parallels the familiar one of the Higgs-Kibble model. Since the potential is invariant under rotations in the $\widetilde{A}_2, \widetilde{B}_2$ plane, we can choose to translate only the field $\widetilde{A}_2$. We write

$$\widetilde{A}_2 \to \widetilde{A}_2 + v \qquad \text{where} \qquad v^2 = 2(\xi g - m^2)/g^2. \tag{9}$$

This translation changes the mass terms in the Lagrangian into the following form

$$-\tfrac{1}{2}(2m^2)(\widetilde{A}_1^2+\widetilde{B}_1^2)-\tfrac{1}{2}(g^2v^2)\widetilde{A}_2^2-\tfrac{1}{2}(g^2v^2)V_\mu^2-\tfrac{1}{2}\mathrm{i}gv\sqrt{2}\,\bar{\lambda}(\gamma_5\psi_2-\psi_1)-\tfrac{1}{2}\mathrm{i}m(\bar{\psi}_1\psi_1+\bar{\psi}_2\psi_2). \tag{10}$$

We see that, as expected, V_μ acquired a mass and $\widetilde{B}_2$ became massless. It is the would-be Goldstone boson of the ordinary gauge invariance, which can be eliminated by the well-known Higgs-Kibble mechanism. However, the spinors now have non-diagonal mass terms which are diagonalized by the following transformation

$$\eta_1 = \tfrac{1}{2}[(1+\cos\beta)\psi_1-(1-\cos\beta)\gamma_5\psi_2-\sqrt{2}\sin\beta\lambda], \quad \eta_2 = \tfrac{1}{2}[(1-\cos\beta)\gamma_5\psi_1+(1+\cos\beta)\psi_2+\sqrt{2}\sin\beta\gamma_5\lambda],$$

$$\zeta = \frac{1}{\sqrt{2}}\sin\beta(\psi_1+\gamma_5\psi_2)+\cos\beta\lambda, \tag{11a, b, c}$$

where $\beta = \arctan(gv/m)$.

With the transformations (11) the fermion mass terms in (10) take the form

$$-\tfrac{1}{2}\mathrm{i}\sqrt{m^2+v^2g^2}(\bar{\eta}_1\eta_1+\bar{\eta}_2\eta_2) \tag{12}$$

which means that the spinor ζ is massless. In fact it is the Goldstone spinor which corresponds to the spontaneously broken supergauge symmetry. Although this is not a surprising result we can still verify it explicitly by studying the transformation properties of the three spinors (11a, b, c) under infinitesimal supergauge transformations[‡4]. A Goldstone field is expected to have in its transformation law a constant term which is not proportional to any other field. For example in a theory containing two real scalar fields φ_1 and φ_2, which is invariant under the rotations $\delta\varphi_1 = \theta\varphi_2$ and $\delta\varphi_2 = \theta\varphi_1$ and in which the symmetry is broken by a translation $\varphi_1 \to \varphi_1 + v$, the Goldstone boson (in the case φ_2) transforms like $\delta\varphi_2 = -\theta(\varphi_1+v)$, i.e., it contains a term $-\theta v$ which is not proportional to any other field of the theory. In our case, we express everything in terms of the original fields and we see that the ones which take a non-zero vacuum expectation value are:

$$\langle A_2\rangle_0 = \langle B_1\rangle_0 = v/\sqrt{2}, \qquad \langle F_2\rangle_0 = \langle G_1\rangle_0 = -mv/\sqrt{2}, \qquad \langle D\rangle_0 = -m^2/g. \tag{13a, b, c}$$

‡4 We thank Dr. B. Zumino for this suggestion as well as for the helpful discussions on this and other questions related to the present investigation.

Using the transformation properties of ref. [1] we find:

$$\delta\psi_1 = -\frac{m v}{\sqrt{2}}\gamma_5\alpha + \ldots\ldots\ldots, \qquad \delta\psi_2 = -\frac{m\dot{v}}{\sqrt{2}}\alpha + \ldots\ldots\ldots, \qquad \delta\lambda = -\frac{m^2}{g}\gamma_5\alpha + \ldots\ldots\ldots \tag{14a, b, c}$$

where the dots stand for the usual terms which are proportional to the other fields. Using the definitions (11) we thus find:

$$\delta\eta_1 = \ldots\ldots\ldots, \qquad \delta\eta_2 = \ldots\ldots\ldots, \qquad \delta\zeta = -\frac{m^2}{g\cos\beta}\gamma_5\alpha + \ldots\ldots\ldots \tag{15}$$

which shows that ζ is indeed a Goldstone spinor.

We shall not give the explicit form of the resulting couplings. The model has obviously no phenomenological interest.

The method presented in this paper can be applied in a straightforward way to a non-abelian model [7] provided the gauge algebra is not semi-simple. The SU(2) × U(1) group, used in the Weinberg-Salam model of weak and electromagnetic interactions, is such an example. But there is no obvious generalization to a semi-simple algebra.

References

[1] J. Wess and B. Zumino, Nucl. Phys. B70 (1974) 39.
[2] S. Ferrara, CERN preprint TH 1824 (1974), Nucl. Phys. B, to be published;
A. Salam and J. Strathdee, Trieste preprint IC/74/11 and IC/74/16;
S. Ferrara, J. Wess and B. Zumino, Phys. Lett. 51B (1974) 239.
[3] J. Iliopoulos and B. Zumino, CERN preprint TH 1834 (1974), Nucl. Phys. B, to be published.
[4] J. Wess and B. Zumino, Phys. Lett. B49 (1974) 52;
S. Ferrara, J. Iliopoulos and B. Zumino, CERN preprint TH 1839 (1974), Nucl. Phys. B., to be published.
[5] A. Salam and J. Strathdee, Trieste preprint IC/74/17.
[6] J. Wess and B. Zumino, CERN preprint TH 1857 (1974), Nucl. Phys. B, to be published.
[7] A. Salam and J. Strathdee, Phys. Lett. 51B (1974) 000;
S. Ferrara and B. Zumino, CERN preprint, TH 1866 (1974).

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SUPERGAUGE INVARIANT EXTENSION OF THE HIGGS MECHANISM AND A MODEL FOR THE ELECTRON AND ITS NEUTRINO

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We extend the Higgs mechanism in a supergauge invariant way. Spontaneous symmetry breaking occurs for all allowed values of the parameters of the scalar potential. With SU(2) × U(1) as gauge group and after spontaneous breaking of supergauge invariance, we obtain a model of weak and electromagnetic interactions for the electron and its neutrino, which includes also heavy leptons and scalar particles. The neutrino is the Goldstone fermion, associated with the photon in a massless vector multiplet.

1. Introduction

Recently, we presented a renormalizable model of weak and electromagnetic interactions in which parity is spontaneously broken, together with SU(2) × U(1) gauge invariance, by the Higgs mechanism [1]. An extra invariance called Q is imposed in order to obtain a massless neutrino. The Higgs field sector is formed by two complex doublets φ' and φ'' exchanged under parity; Q-invariance constrains the potential in such a way that one, and only one, of them takes a non-zero vacuum expectation value. While SU(2) × U(1) gauge invariance is reduced down to the usual electromagnetic one, parity – still conserved in electromagnetism – is broken in a maximal way in classical weak interactions.

In sect. 2, we present a model very similar** to that of ref. [1], describing the gauge-, parity-, and Q-invariant interaction of the gauge vector bosons W_μ and B_μ, the Higgs doublets φ' and φ'', the doublet ψ of Dirac spinors and the singlet Majorana spinor λ. After spontaneous breaking the lepton sector contains the electron, its left-handed neutrino and a Dirac heavy neutrino.

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** The only difference is that here we use a Majorana spinor λ instead of the Dirac spinor S of ref. [1], as it had been suggested by Bouchiat. Therefore the right-handed neutrino field present in ref. [1] (but only weakly coupled through the Higgs fields) does not appear.

But why do we impose Q-invariance to the model? In fact, the real question is: why is there a massless fermion, the neutrino, in nature? A possible answer could be to consider the neutrino as a Goldstone particle [2] associated with the spontaneous breaking of a supersymmetry, for which the parameters of infinitesimal transformations are totally anticommuting Majorana spinors. Such a symmetry ("supergauge symmetry" or "Fermi-Bose supersymmetry") has been introduced by Wess and Zumino [3,4], and Iliopoulos and the author have shown that it could be spontaneously broken, thus producing a Goldstone spinor [5]. The breakdown of supergauge invariance allows us to construct theories in which the masses of fermions are not systematically degenerated with those of bosons.

In sect. 3, we first recall the supergauge invariant extension of quantum electrodynamics of Wess and Zumino [4]. The vector multiplet contains a vector W_μ and a Majorana spinor λ; the complex scalar multiplet contains a Dirac spinor ψ and two complex scalars φ' and φ'' exchanged under parity. The Lagrangian density is very similar to that of sect. 2: if we had started with U(1) as the gauge group instead of SU(2) × U(1), we would have found the model of ref. [4] with a suitable choice of parameters. This last model is therefore invariant under Q-transformations which have a very simple definition in terms of the superfields introduced by Salam and Strathdee [6,7].

No spontaneous symmetry-breaking occurs in [4]. In [5] a supergauge-, gauge-, and Q-invariant term is added to the action integral, which breaks parity explicitly but softly. It triggers a spontaneous breaking of supergauge- and gauge invariance, and, owing to the Q -invariant structure of the potential, one and only one of the fields φ' and φ'' takes a non-zero vacuum expectation value.

In sect. 4, we come back to SU(2) × U(1) models: to the model of sect. 2 we add a triplet of Majorana spinors λ, and associate λ with W_μ, λ with B_μ in order to form vector multiplets of "generalized gauge fields". They interact with the doublet of complex scalar multiplets $(\psi, \varphi', \varphi'')$. A spontaneous breaking of supergauge- and gauge-invariance is obtained, but the broken theory is not an acceptable weak interaction model: the "electron" would be massless.

In sect. 5, we add a gauge-invariant scalar multiplet $\mathcal{O}$ to the previous model, and demand the theory to be invariant under a group of R-transformations. If we impose also parity-invariance, supergauge-invariance is conserved while gauge-invariance is spontaneously broken. Three of the vector multiplets become massive while three real scalar multiplets are eliminated by a "super-Higgs mechanism". The corresponding model with U(1) as the gauge group is just the supergauge-invariant extension of the Higgs-Kibble model [8]. The advantage here is that the potential is always positive and necessarily induces spontaneous breaking of gauge invariance. The model, still supergauge-invariant, contains the massless vector multiplet $(\gamma; \lambda_\gamma)$, the massive ones $(Z; E_0; z)$ and $(W_-; E_-, e_-; w_-)$ together with a complex scalar multiplet $(e_0; \omega, \varphi)$.

Supergauge-invariance is spontaneously broken as in [5]; λ_γ associated with the photon in a massless vector multiplet, is the Goldstone spinor. The equality of masses

in multiplets is preserved, at least at the lowest order, except for the e_ and E_ masses split on each side of the W_ or w_ masses. The charged vector currents coupling λ_γ to e_ and E_ are left-handed and right-handed respectively. $\nu_L = \frac{1}{2}(1 - i\gamma_5)\lambda_\gamma$ is interpreted as the neutrino field; e_ and E_ are the electron and heavy electron respectively; e_0 and E_0 are two heavy neutrinos.

R-invariance, conserved through spontaneous symmetry breaking, is associated with lepton-number conservation; e_, e_0 and ν_L have lepton number +1; E_ and E_0 −1; a lepton number +2 is attributed to the complex scalar ω.

When re-written in terms of physical fermion fields, the doublet of Dirac spinors ψ and the triplet of Majorana spinors λ depend only on the two angles θ and δ. θ is at the same time the SU(2) × U(1) mixing angle of Weinberg-Salam-type models [9], and the neutrino-heavy-neutrino mixing angle of Georgi-Glashow-type models [10]. The model provides a solution for the problem of Fermion anomalous magnetic moment in supergauge theories, which has been shown [11] to vanish in the model of ref. [4]. Here we find $\alpha/2\pi$ for the electron anomaly at the one-loop level, with negligible weak corrections.

However, there seems to be no simple way to extend the model in order to incorporate the muon, since a massless vector multiplet contains a vector but only one Majorana spinor: we have associated the photon with ν_e which is the Goldstone particle, but we cannot obtain a massless ν_μ in the same way.

2. A gauge theory of weak and electromagnetic interactions with spontaneous parity-breaking

2.1. Lagrangian density and symmetry-breaking

As indicated before, the model we describe here is similar to the model of ref. [1], except that a Majorana spinor λ is used as the SU(2) singlet instead of the Dirac spinor S of ref. [1]. The gauge group is SU(2) × U(1). The couplings of the gauge vector bosons W_μ and B_μ to T and F, the generators of SU(2) and U(1) respectively, are determined by the expression* of the covariant derivative:

$$iD_\mu = i\partial_\mu - (gTW_\mu + \tfrac{1}{2}g'FB_\mu)\,. \tag{1}$$

The Majorana spinor λ is an SU(2) × U(1) singlet; ψ is an SU(2) doublet of Dirac spinors, which has the U(1) quantum number $F = 1$, and whose charged member will be the electron field. φ' and φ'' are Higgs doublets exchanged under parity, which transform like ψ under the gauge group. The Lagrangian density must contain

* Notice that g', and then the mixing angle θ, have opposite signs as in ref. [1]. $e = \sqrt{g^2 + g'^2}$ $\sin\theta\cos\theta$ is here the electron charge ($e < 0$). The sign of the metric also is different.

a $-im_e\bar{\psi}\psi$* mass term for the electron to be massive. In order to have a massless fermion (which is necessarily λ when the symmetry is unbroken), we must not have a $-\frac{1}{2}im_\lambda\bar{\lambda}\lambda$ mass term. To avoid it we are led, as in ref. [1], to impose invariance under the group of Q-transformations:

$$Q\,\psi\,Q^{-1} = \psi\,, \qquad Q\,\varphi'\,Q^{-1} = e^{-i\alpha}\varphi'\,,$$

$$Q\,\lambda\,Q^{-1} = e^{\gamma_5\alpha}\lambda\,, \qquad Q\,\varphi''\,Q^{-1} = e^{i\alpha}\varphi''\,. \tag{2}$$

The most general gauge-, parity- and Q-invariant Lagrangian density reads:

$$\begin{aligned}\mathcal{L} = {} & -\tfrac{1}{4}B_{\mu\nu}B^{\mu\nu} \quad \tfrac{1}{4}W_{\mu\nu}W^{\mu\nu} - i\,\bar{\psi}(\not{D} + m_e)\,\psi - \tfrac{1}{2}i\bar{\lambda}\not{\partial}\lambda \\ & + ih\sqrt{2}[(\bar{\psi}_R\varphi' + \bar{\psi}_L\varphi'')\lambda + \bar{\lambda}(\varphi'^\dagger\psi_R + \varphi''^\dagger\psi_L)] \\ & - (D_\mu\varphi'^\dagger D^\mu\varphi' + D_\mu\varphi''^\dagger D^\mu\varphi'') - \mu^2(\varphi'^\dagger\varphi' + \varphi''^\dagger\varphi'') \\ & - b[(\varphi'^\dagger\varphi')^2 + (\varphi''^\dagger\varphi'')^2] - c\varphi'^\dagger\varphi'\varphi''^\dagger\varphi'' - d\varphi'^\dagger\varphi''\varphi''^\dagger\varphi'\,,\end{aligned} \tag{3}$$

where we have denoted the $\frac{1}{2}(1 - i\gamma_5)$ and $\frac{1}{2}(1 + i\gamma_5)$ parts of the Dirac fields by the indices L and R respectively. Q-invariance restricts the possible terms in the Lagrangian considerably. Spontaneous symmetry breaking occurs (see appendix of ref. [1]) if

$$\mu^2 < 0, \qquad b > 0, \qquad c - 2b > \mathrm{Sup}(-d, 0)\,. \tag{4}$$

Then one, and only one, of the fields φ' and φ'' takes a non-zero vacuum expectation value; we choose for example φ', and we have, in the convenient gauge**:

$$\langle\varphi'\rangle = \begin{bmatrix} v/\sqrt{2} \\ 0 \end{bmatrix} \quad \text{with} \quad v^2 = -\frac{\mu^2}{b}\,. \tag{5}$$

The mass terms of the spinor fields:

$$\mathcal{L}_m = -im_e\bar{\psi}\psi + ihv(\bar{\psi}_{uR}\lambda + \bar{\lambda}\psi_{uR}) \tag{6}$$

are diagonalized by the transformations:

$$\begin{aligned}\nu' &= (\psi_u\cos\beta - \lambda\sin\beta)_L + \psi_{uR}\,, \\ \nu_L &= (\psi_u\sin\beta + \lambda\cos\beta)_L\,, \\ e &= \psi_d\,,\end{aligned} \tag{7}$$

* We use the Majorana representation of γ matrices, where the γ are real and satisfy:

$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}$ with $g_{00} = -1$,

$\gamma_5 = -\gamma_0\gamma_1\gamma_2\gamma_3$ and $\tilde{\gamma}_\mu = -\gamma_\mu$. $\gamma_5^2 = 1$.

The main properties of γ matrices can be found in ref. [3].

** In ref. [1] we defined $\langle\varphi'_0\rangle = v$.

where

$$\mathrm{tg}\,\beta = h v/m_e\,.$$

The use of Dirac spinors is justified by the invariance of the broken theory under a group of X-transformations defined by

$$X\,\psi\,X^{-1} = e^{i\alpha}\psi\,, \qquad X\,\varphi'\,X^{-1} = \varphi'\,,$$

$$X\,\lambda\,X^{-1} = e^{\gamma_5\alpha}\lambda\,, \qquad X\,\varphi''\,X^{-1} = e^{2i\alpha}\varphi''\,. \tag{8}$$

Under an X-transformation ν', ν_L and e are multiplied by $e^{i\alpha}$, and φ'' by $e^{2i\alpha}$. X-invariance is associated with lepton number conservation: ν', ν_L and e have lepton number +1 while φ'' has lepton number +2 (φ''_0 and φ''_- are two heavy scalars). ν_L is identified with the neutrino field; its mass vanishes identically. e and ν' are the electron and heavy neutrino fields respectively; their masses are connected through the zeroth-order relation:

$$m_e = m_\nu \cos\beta \tag{9}$$

which forces the angle β to be near $\frac{1}{2}\pi$.

2.2. Consequences

The same formula as in ref. [1] expresses ψ in terms of the physical fermion fields:

$$\psi = \begin{bmatrix} \nu_L \sin\beta + \nu'_L \cos\beta + \nu'_R \\ \\ e \end{bmatrix}. \tag{10}$$

The couplings of the gauge vector bosons are then identical in both models. To introduce the muon sector, we just double the spinor fields ψ and λ*. The experimental consequences of (9) and (10) have been studied in detail in refs. [1] and [12]. The essential feature of the weak neutral current in such a model is that its electronic or muonic parts are parity-conserving. Equal $\nu_\mu e$ and $\bar\nu_\mu e$ cross sections are then predicted:

$$\sigma_{\nu_\mu e} = \sigma_{\bar\nu_\mu e} = \sigma^{V-A}_{\bar\nu_e e}\,(2\sin^2\theta - 1)^2\,, \tag{11}$$

where θ (defined by $\mathrm{tg}\,\theta = g'/g$) is the usual SU(2) × U(1) mixing angle [9]. Present experimental data available for $\nu_\mu e$, $\bar\nu_\mu e$, and $\bar\nu_e e$ scatterings [13,14] are compatible with

* β_e and β_μ are the two mixing angles, not necessarily equal; however we have $\sin\beta_e \simeq \sin\beta_\mu \simeq 1$, which ensures e–$\mu$ universality with deviations $\sim (m_e/m_{\nu' e})^2$ or $(m_\mu/m_{\nu' \mu})^2$.

$$0.17 < \sin^2\theta < 0.39,$$

or

$$0.61 < \sin^2\theta < 0.70. \tag{12}$$

However, the present model differs from the model of ref. [1] when scalar boson exchanges are considered. In ref. [1], the use of Dirac spinors gave separate electronic and muonic number conservation automatically. Here X-invariance leads only to total leptonic number conservation. Electronic and muonic numbers are separately conserved in vector boson exchanges but not in scalar ones; we find for example a very small coupling between the left-handed electron field and the right-handed antineutrino field, given by

$$\mathcal{L} = \frac{ih}{\sqrt{2}}\cos\beta\,[\bar e(1+i\gamma_5)\nu^c\varphi''_- + \varphi''^{\dagger}_-\bar\nu^c(1-i\gamma_5)e]\,. \tag{13}$$

In addition to the usual μ-decay amplitude for ($\mu \to \nu_{\mu L} + e + \bar\nu_{eR}$), an extra amplitude for ($\mu \to \bar\nu_{\mu R} + e + \nu_{eL}$), due to a φ''_- exchange, is obtained, and is given by the effective Lagrangian density:

$$\mathcal{L}_{\mathrm{eff}} = \frac{G}{\sqrt{2}}\frac{m_\mu m_e}{m^2_{\varphi''_-}}\bar e(1+i\gamma_5)\nu^c_e\,\bar\nu^c_\mu(1-i\gamma_5)\mu\,. \tag{14}$$

Such an amplitude is completely negligible, whatever reasonable value is chosen for $m_{\varphi''}$. On the contrary, in ref. [1], φ''_- exchange induced an amplitude $\mu \to \nu_{\mu R} + e + \bar\nu_{eL}$ (conserving separately electronic and muonic numbers), but which was negligible only if φ''_- was sufficiently heavy.

The present model of leptons provides a good description of the phenomenology of weak and electromagnetic interactions; and can be regarded as a possible alternative to the Weinberg model [9]. At the price of introducing two heavy neutrinos ν'_e and ν'_μ, the model shows how to obtain both an electromagnetism which conserves parity, and classical weak interactions which violate it in a maximal way. It predicts very small violations of separate electronic and muonic number conservation.

In the next section we will exhibit close similarities between the above model and supergauge theories: the supergauge-invariant extension of quantum electrodynamics of Wess and Zumino [4] and the model derived by Iliopoulos and the author, where a spontaneous breaking of supergauge- and gauge-invariance is obtained. $\mathcal{Q}$-invariance, which can still be defined for these models, will be expressed in a very simple way in terms of superfields.

3. Connection with supergauge invariant extensions of quantum electrodynamics

3.1. Supergauge invariant extension of quantum electrodynamics

Let us first recall the model of ref. [4]. It describes the supergauge invariant interaction of a complex scalar supergauge multiplet $\mathcal{S}$ with a vector supergauge multiplet of gauge fields. The complex scalar $\mathcal{S} = \sqrt{\frac{1}{2}}(\mathcal{S}_1 + i\mathcal{S}_2)$ contains a Dirac spinor ψ, a complex scalar A, and a complex pseudoscalar B, together with the auxiliary fields F (scalar) and G (pseudoscalar). The vector multiplet $\mathcal{V}$ contains a vector W_μ, a Majorana spinor λ and several auxiliary fields (among them the pseudoscalar D). Gauge transformations, as usually defined, are not compatible with supergauge invariance: generalized U(1) gauge transformations are defined, for which the role of the gauge function is played by a whole scalar multiplet, and the components of $\mathcal{V}$ are the (generalized) gauge fields. λ and D are invariant under a generalized gauge transformation whereas W_μ is changed by a gradient.

$\mathcal{S}$ and $\mathcal{S}^\dagger$ can be combined into vector multiplets using the symmetric or antisymmetric combinations laws denoted by $\times$ and $\wedge$ respectively. The internal symmetric combination laws of vector, or scalar, multiplets, are also used, together with the scalar product of two vector multiplets. The Lagrangian density is constructed as a sum of D components of vector multiplets or F components of scalar multiplets: under a supergauge transformation, they are invariant up to a four-derivative, and the corresponding action integral is invariant.

The most general Lagrangian density, invariant under supergauge*, gauge and parity transformations, can be written in the following form:

$$\mathcal{L} = \tfrac{1}{2}[\mathcal{S} \times \mathcal{S}^\dagger \operatorname{ch} 2g\mathcal{V} + i\,\mathcal{S} \wedge \mathcal{S}^\dagger \operatorname{sh} 2g\mathcal{V}]_D + m[\mathcal{S}^\dagger \mathcal{S}]_F - \tfrac{1}{2}(\mathcal{V}, \mathcal{V})_F , \tag{15}$$

where $\frac{1}{2}[\mathcal{S} \times \mathcal{S}^\dagger]_D$ and $-\frac{1}{2}(\mathcal{V}, \mathcal{V})_F$ are respectively the kinetic energy terms of the scalar and vector multiplets.

Performing a generalized gauge transformation, one can choose the gauge ("special gauge") in order for W_μ, λ and D to be the only components of $\mathcal{V}$ which do not vanish. Instead of the scalar A and the pseudoscalar B, we use the complex fields φ' and φ'' exchanged under parity:

$$\varphi' = -\sqrt{\tfrac{1}{2}}\, i(A + iB) ,$$
$$\varphi'' = -\sqrt{\tfrac{1}{2}}\, i(A - iB) . \tag{16}$$

The Lagrangian density (15) in the special gauge and after elimination of auxiliary fields reads:

* From now on we omit to precise "up to a four-derivative".

$$\mathcal{L} = -\tfrac{1}{4} W_{\mu\nu} W^{\mu\nu} - \tfrac{1}{2} i\bar{\lambda}\partial\!\!\!/\lambda - i\bar{\psi}(\partial\!\!\!/ + m)\psi + ig\sqrt{2}[(\bar{\psi}_R \varphi' + \bar{\psi}_L \varphi'')\lambda + \bar{\lambda}(\varphi'^\dagger \psi_R + \varphi''^\dagger \psi_L)] - (D_\mu \varphi'^\dagger D^\mu \varphi' + D_\mu \varphi''^\dagger D^\mu \varphi'') - m^2(\varphi'^\dagger \varphi' + \varphi''^\dagger \varphi'') - \tfrac{1}{2} g^2 (\varphi'^\dagger \varphi' - \varphi''^\dagger \varphi'')^2 . \tag{17}$$

It looks like the Lagrangian density (3) of the model described in sect. 2. The latter was however an SU(2) × U(1) model; the U(1) gauge invariant model analogous to it corresponds to the Lagrangian density:

$$\mathcal{L} = -\tfrac{1}{4} W_{\mu\nu} W^{\mu\nu} - \tfrac{1}{2} i\bar{\lambda}\partial\!\!\!/\lambda - i\bar{\psi}(\partial\!\!\!/ + m)\psi + ih\sqrt{2}[(\bar{\psi}_R \varphi' + \bar{\psi}_L \varphi'')\lambda + \bar{\lambda}(\varphi'^\dagger \psi_R + \varphi''^\dagger \psi_L)] - (D_\mu \varphi'^\dagger D^\mu \varphi' + D_\mu \varphi''^\dagger D^\mu \varphi'') - \mu^2(\varphi'^\dagger \varphi' + \varphi''^\dagger \varphi'') - b[(\varphi'^\dagger \varphi')^2 + (\varphi''^\dagger \varphi'')^2] - c\,\varphi'^\dagger \varphi' \varphi''^\dagger \varphi'' . \tag{18}$$

We see that (17) is a special case of (18); therefore the model of ref. [4] is Q-invariant, with Q-invariance still defined by (2). Supergauge invariance imposes the further constraints:

$$\mu = m ,$$
$$h = g , \qquad b = \tfrac{1}{2} g^2 , \qquad c = -g^2 . \tag{19}$$

It fixes equal masses for the scalar fields φ' and φ'' and the Dirac field ψ; it determines the coupling constant of scalar fields with fermion fields, and the potential of scalar fields in terms of the gauge coupling constant g.

No spontaneous symmetry-breaking occurs in this model. In ref. [5] Iliopoulos and the author have added to the Lagrangian density (15) the term ξD which is gauge invariant, and supergauge invariant up to a four derivative; this term breaks parity explicitly but softly.

Then in formula (17) we add the term:

$$\xi g(\varphi'^\dagger \varphi' - \varphi''^\dagger \varphi'') , \tag{20}$$

which is also Q invariant. Supergauge invariance is spontaneously broken as soon as $\xi g \neq 0$, together with U(1) gauge invariance if $\xi g \nless m^2$*. Because of the Q-invariance of the potential only φ' takes a non-zero vacuum expectation value $v/\sqrt{2}$ given by:

$$\tfrac{1}{2} g^2 v^2 = \xi g - m^2 . \tag{21}$$

* We choose $\xi g > 0$ without loss of generality.

A Goldstone fermion appears [2,15], described by the two-component spinor:

$$\chi_2 = \tfrac{1}{2}(1 - i\gamma_5)[\psi \sin\beta + \lambda \cos\beta] \tag{22}$$

with $\mathrm{tg}\,\beta = gv/m$. Its mass vanishes as a consequence of Goldstone theorem; this can also be shown directly, exactly as in sect. 2.

3.2. Q invariance and superfields

The fields φ', φ'' and ψ appearing in refs. [1,4,5] are the components of a complex scalar multiplet, and it is remarkable that the Q-transformations defined by formula (2), first exhibited in ref. [1], can be interpreted as transformations on superfields [illegible] the left-handed superfields defining the real scalar multiplets $\mathcal{O}_1$

[illegible] (24)

[illegible] the transformation [illegible] first term in the Lagrangian density (15) (kinetic [illegible] and interaction terms with the generalized gauge [illegible] $x, \theta, \bar\theta$) deduced from $\mathcal{V}(x, \theta, \bar\theta)$ by the use of conjugate to θ and $\bar\theta$, and a translation on x (see ref. [17]) are transformed like $\mathcal{O}_i$ under Q. Both $\mathcal{W}^2$ and $(\mathcal{O}_1^2 + \mathcal{O}_2^2)$ have Q-invariant F-components. The free Lagrangian density of the U(1) gauge vector multiplet* and the mass term of the scalar multiplet are Q-invariant.

The whole Lagrangian density (15) is therefore Q-invariant, together with the parity-breaking term ξD which can be added to it. Q-invariance of models of refs. [4,5] exhibited in the previous subsection is found here only by means of transformations on superfields.

We now come back to SU(2) × U(1) gauge theories, and in the next section we construct the supergauge invariant Lagrangian density associated with the model described in sect. 2.

4. A first attempt for an SU(2) × U(1) supergauge model

Salam and Strathdee [16], and Ferrara and Zumino [17] have succeeded in constructing theories which are invariant both under supergauge and non-Abelian gauge transformations. The simplest is the ordinary Yang-Mills theory of vectors in interaction with a multiplet of massless Majorana spinors belonging to the regular representation of the internal symmetry group. We know now that supergauge invariance can be spontaneously broken, and the method used in ref. [5] can be applied to any theory in which the gauge group is not semi-simple, the simplest example being SU(2) × U(1).

We recall the fields appearing in the SU(2) × U(1) model of sect. 2: two Higgs doublets φ' and φ'' exchanged under parity; the doublet ψ of Dirac spinors; the U(1) gauge singlet B_μ and the SU(2) gauge triplet W_μ. We showed in sect. 3 that the U(1) version of this model was supergauge invariant for special values of the parameters, and we identified it with the supergauge invariant extension of quantum electrodynamics of ref. [4]. φ', φ'' and ψ form a complex scalar multiplet while λ and B_μ form a vector multiplet. Supergauge invariance for an SU(2) × U(1) gauge theory leads to the introduction of a triplet λ of Majorana spinors associated with W_μ to form a [illegible] multiplet. The supergauge [illegible] SU(2) × U(1) invariant interaction of the SU(2) doublet of scalar multiplets $\mathcal{D}$ with the triplet $\mathcal{V}$ and the singlet $\mathcal{V}'$ of vector multiplets of gauge fields. These are written as 2 × 2 matrices:

$$\mathcal{V} = \begin{pmatrix} \mathcal{V}_3 & \mathcal{V}_1 - i\mathcal{V}_2 \\ \mathcal{V}_1 + i\mathcal{V}_2 & -\mathcal{V}_3 \end{pmatrix}, \qquad \mathcal{V}' = \begin{pmatrix} 0 & \mathcal{V}' \\ \mathcal{V}' & 0 \end{pmatrix}$$

Similar notations are used for their components. For commodity we call W'_μ and λ' (instead of B_μ and λ) the physical components of the vector multiplet $\mathcal{V}'$. Formula (15) with the parity-breaking term ξD is now replaced by:

$$\mathcal{L} = \xi [\mathcal{V}']_D + [\mathcal{D}^\dagger e^{g\mathcal{V} + g'\mathcal{V}'} \mathcal{D}]_D + m[\mathcal{D}^\dagger \mathcal{D}]_F + \mathcal{L}_0 + \xi[\mathcal{V}']_D \tag{26}$$

where $\mathcal{L}_0$ is the Lagrangian density of the free vector multiplet $\mathcal{V}'$ and the self-interacting vector multiplets $\mathcal{V}$.

Performing an SU(2) × U(1) generalized gauge transformation we choose a gauge where all components of the vector multiplets vanish except W_μ, λ, D and W'_μ, λ', D'. The only term surviving in $(g\mathcal{V} + g'\mathcal{V}')^2$ is the D component which equals $-(gW_\mu + g'W'_\mu)^2$, while the higher powers $(g\mathcal{V} + g'\mathcal{V}')^n$ $(n > 2)$ vanish identically. Expression (26) in the special gauge is quadratic in g and g'.

Using the definitions:

$$\varphi' = -\sqrt{\tfrac{1}{2}}\, i(A + iB), \qquad H' = -\sqrt{\tfrac{1}{2}}\, i(F + iG),$$

$$\varphi'' = -\sqrt{\tfrac{1}{2}}\, i(A - iB), \qquad H'' = -\sqrt{\tfrac{1}{2}}\, i(F - iG), \tag{27}$$

and the expression (1) of the covariant derivative D_μ, we find:

$$\mathcal{L} = -\tfrac{1}{4} W_{\mu\nu} W^{\mu\nu} - \tfrac{1}{2} i\bar\lambda \not{D} \lambda + \tfrac{1}{2} D^2$$

* The same argument holds for the Lagrangian density of self-interacting gauge vector multiplets in a Yang-Mills theory.

$$-\tfrac{1}{4}W'_{\mu\nu}W'^{\mu\nu} - \tfrac{1}{2}i\bar{\lambda}'\partial\!\!\!/\lambda' + \tfrac{1}{2}D'^2 + \xi D'$$

$$+ (H'^{\dagger}H' + H''^{\dagger}H'' - D_\mu\varphi'^{\dagger}D^\mu\varphi' - D_\mu\varphi''^{\dagger}D^\mu\varphi'' - i\bar{\psi}\partial\!\!\!/\psi)$$

$$+ m(H'^{\dagger}\varphi' + \varphi'^{\dagger}H' + H''^{\dagger}\varphi'' + \varphi''^{\dagger}H'' - i\bar{\psi}\psi)$$

$$+ \sqrt{\tfrac{1}{2}}i[\bar{\psi}_R(g\tau\lambda + g'\lambda')\varphi' + \varphi'^{\dagger}(g\tau\bar{\lambda} + g'\bar{\lambda}')\psi_R$$

$$+ \bar{\psi}_L(g\tau\lambda + g'\lambda')\varphi'' + \varphi''^{\dagger}(g\tau\bar{\lambda} + g'\bar{\lambda}')\psi_L]$$

$$+ \tfrac{1}{2}[\varphi''^{\dagger}(g\tau D + g'D')\varphi'' - \varphi'^{\dagger}(g\tau D + g'D')\varphi'] \,. \qquad (28)$$

Supergauge invariance is spontaneously broken for $\xi \neq 0$, together with gauge invariance if $|\frac{1}{2}\xi g'| > m^2$. Then φ' (but not φ'') takes a non-vanishing vacuum expectation value $\begin{bmatrix} v'/\sqrt{2} \\ 0 \end{bmatrix}$ with:

$$\tfrac{1}{8}(g^2 + g'^2)v'^2 = \tfrac{1}{2}\xi g' - m^2 \,. \qquad (29)$$

The broken theory appears to contain two massless Dirac spinors, one charged, and one neutral, and thus cannot receive a physical interpretation*. In the next section, we modify the model in order for the only massless particles to be the photon and the neutrino (Goldstone spinor) associated in a vector multiplet.

5. A supergauge model of weak and electromagnetic interactions of the electron and its neutrino

5.1. R Invariance and Lagrangian density

We introduce a gauge invariant scalar multiplet $\mathcal{S}$, with components a, b, p, f_0, g_0, in the model**. In addition to supergauge and gauge invariance, we demand the invariance under a group of R-transformations, which at the end will be identified with the lepton number conservation group. With notations similar to those of sect. 3.2 we define the action of R on superfields:

$$\begin{aligned}
&R\,\underline{\mathcal{V}}\;(x,\theta,\bar{\theta})\,R^{-1} = \underline{\mathcal{V}}\;(x,\theta e^{-i\alpha},\bar{\theta}e^{i\alpha}) \,,\\
&R\,\underline{\mathcal{V}'}\;(x,\theta,\bar{\theta})\,R^{-1} = \underline{\mathcal{V}'}\;(x,\theta e^{-i\alpha},\bar{\theta}e^{i\alpha}) \,,\\
&R\,\underline{\mathcal{D}_{ki}}(x,\theta)\;\;R^{-1} = \underline{\mathcal{D}_{ki}}(x,\theta e^{-i\alpha}) \,,\\
&R\,\underline{\mathcal{S}}\;\;(x,\theta)\;\;R^{-1} = e^{2i\alpha}\,\underline{\mathcal{S}}\,(x,\theta e^{-i\alpha}) \,,
\end{aligned} \qquad (30)$$

* If the electron was massless, the two Dirac spinors could be interpreted as electron and neutrino fields; the V−A expression of charged currents would be obtained for small values of the mixing angle θ.

** We use the index 0 only in order to distinguish the field g_0 from the coupling constant g.

where the $\mathcal{D}_{ki}$ (k = u, d; i = 1,2) are the four left-handed superfields associated with the complex doublet of scalar multiplet $\mathcal{D}$.

The Lagrangian density (26) is R-invariant for $m = 0$. The only terms depending on $\mathcal{S}$, of dimension less or equal to four, and supergauge, gauge, R- and parity-invariant are the kinetic energy term of $\mathcal{S}$ and the F component of the multiplets $\mathcal{S}$ and $\mathcal{D}^\dagger\mathcal{D}\mathcal{S}$. We must also use the parity breaking terms of dimension $\leqslant 2$, i.e. the G-component of the multiplet $\mathcal{S}$. Then to the Lagrangian density (26) with $m = 0$, we add the term:

$$s[f_0\cos u + g_0 \sin u] + \tfrac{1}{2}h[\mathcal{D}^\dagger\mathcal{D}\mathcal{S}]_F \,, \qquad (31)$$

where u is defined so that $s/h < 0$.

We extend the definition (24) of Q-transformations to $\underline{\mathcal{S}}$, in which $\underline{\mathcal{V}}$ has been replaced by $\underline{\mathcal{V}}$ or $\underline{\mathcal{V}'}$, $\underline{\mathcal{S}_i}$ by $\underline{\mathcal{D}_{ki}}$:

$$Q\,\underline{\mathcal{S}}(x,\theta)\,Q^{-1} = \underline{\mathcal{S}}(x,\theta e^{-i\alpha}) \qquad (32)$$

and define the product $U = Q\,R^{-1}$ which acts in multiplying the left-handed superfields $\underline{\mathcal{D}_{ki}}$ and $\underline{\mathcal{S}}$ by $e^{i\alpha}$ and $e^{-2i\alpha}$ respectively. All terms in the Lagrangian density are U-invariant with the exception of $s(f_0\cos u + g_0 \sin u)$. Thus this term can be cast into the simpler form $s f_0$ if convenient phase transformations for $\underline{\mathcal{D}_{ki}}$ and $\underline{\mathcal{S}}$ are performed.

The whole Lagrangian density is now:

$$\mathcal{L} = \tfrac{1}{2}\mathrm{Tr}[\mathcal{D}\times\mathcal{D}^\dagger \mathrm{ch}(g\mathcal{V} + g'\mathcal{V}') + i\,\mathcal{D}\wedge\mathcal{D}^\dagger \mathrm{sh}(g\mathcal{V} + g'\mathcal{V}')]_D + \mathcal{L}_0 + \xi D'$$

$$+ \tfrac{1}{4}[\mathcal{S}\times\mathcal{S}]_D + s f_0 + \tfrac{1}{2}h[\mathcal{D}^\dagger\mathcal{D}\mathcal{S}]_F \,, \qquad (33)$$

It reads in the special gauge*:

$$\begin{aligned}
\mathcal{L} = {} & [-\tfrac{1}{4}W_{\mu\nu}W^{\mu\nu} - \tfrac{1}{2}i\bar{\lambda}\partial\!\!\!/\lambda + \tfrac{1}{2}D^2] + [-\tfrac{1}{4}W'_{\mu\nu}W'^{\mu\nu} - \tfrac{1}{2}i\bar{\lambda}'\partial\!\!\!/\lambda' + \tfrac{1}{2}D'^2 + \xi D']\\
& + [H'^{\dagger}H' + H''^{\dagger}H'' - D_\mu\varphi'^{\dagger}D^\mu\varphi' - D_\mu\varphi''^{\dagger}D^\mu\varphi'' - i\bar{\psi}\partial\!\!\!/\psi]\\
& + i\sqrt{\tfrac{1}{2}}[\psi_R(g\tau\lambda + g'\lambda')\varphi' + \varphi'^{\dagger}(g\tau\bar{\lambda} + g'\bar{\lambda}')\psi_R\\
& + \bar{\psi}_L(g\tau\lambda + g'\lambda')\varphi'' + \varphi''^{\dagger}(g\tau\bar{\lambda} + g'\bar{\lambda}')\psi_L]\\
& + \tfrac{1}{2}[\varphi''^{\dagger}(g\tau D + g'D')\varphi'' - \varphi'^{\dagger}(g\tau D + g'D')\varphi']\\
& + \tfrac{1}{2}[f_0^2 + g_0^2 - \partial_\mu a\,\partial^\mu a - \partial_\mu b\,\partial^\mu b - i\bar{p}\,\partial\!\!\!/\,p] + s f_0\\
& + \tfrac{1}{2}h\{(f_0 + ig_0)\varphi'^{\dagger}\varphi'' + (f_0 - ig_0)\varphi''^{\dagger}\varphi'\\
& + (a + ib)(H'^{\dagger}\varphi' + \varphi''^{\dagger}H'') + (a - ib)(H''^{\dagger}\varphi'' + \varphi'^{\dagger}H')\\
& - i\bar{\psi}(a - \gamma_5 b)\psi + \sqrt{2}\,[(\bar{\psi}_R\varphi'' + \bar{\psi}_L\varphi')p - \bar{p}(\varphi''^{\dagger}\psi_R + \varphi'^{\dagger}\psi_L)]\}
\end{aligned} \qquad (34)$$

* This Lagrangian density is expected to give a theory renormalizable in a way compatible with supergauge and R invariances.

The auxiliary fields D, D', H', H'', f_0, g_0 are eliminated by means of the equations of motion. This leads to the potential:

$$V(\varphi', \varphi'', a, b) = \tfrac{1}{2}g^2(\varphi'^\dagger\varphi'\varphi''^\dagger\varphi'' - \varphi'^\dagger\varphi''\varphi''^\dagger\varphi')$$
$$+ \tfrac{1}{8}(g^2+g'^2)(\varphi''^\dagger\varphi'' - \varphi'^\dagger\varphi')^2 + \tfrac{1}{2}\xi g'(\varphi''^\dagger\varphi'' - \varphi'^\dagger\varphi')$$
$$+ \tfrac{1}{4}h^2(a^2+b^2)(\varphi'^\dagger\varphi' + \varphi''^\dagger\varphi'') + \tfrac{1}{2}|s + h\varphi''^\dagger\varphi'|^2 . \qquad (35)$$

5.2. Spontaneous symmetry breaking

The equations fixing the minimum of the potential are:

$$\varphi'^\dagger\varphi'\varphi''^\dagger\varphi'' - \varphi'^\dagger\varphi''\varphi''^\dagger\varphi' = 0 ,$$
$$\tfrac{1}{4}(g^2+g'^2)(\varphi''^\dagger\varphi'' - \varphi'^\dagger\varphi') + \tfrac{1}{2}\xi g' = 0 ,$$
$$a = b = 0 , \qquad s + h\varphi''^\dagger\varphi' = 0 . \qquad (36)$$

The first one implies that φ' and φ'' are proportional, and the last one that their relative phase vanishes. In the convenient gauge φ' and φ'' take the vacuum expectation values

$$\langle\varphi'\rangle = \begin{bmatrix} v'/\sqrt{2} \\ 0 \end{bmatrix} , \qquad \langle\varphi''\rangle = \begin{bmatrix} v''/\sqrt{2} \\ 0 \end{bmatrix} , \qquad (37)$$

where the real v' and v'' are fixed by*:

$$\tfrac{1}{8}(g^2+g'^2)(v'^2 - v''^2) = \tfrac{1}{2}\xi g' ,$$
$$s + \tfrac{1}{2}hv'v'' = 0 . \qquad (38)$$

For any non-vanishing s and h, both v' and v'' are non-zero (contrary to the situation in the previous models). Gauge invariance is spontaneously broken.

In analogy with:

$$Z^\mu = W_3^\mu \cos\theta + W'^\mu \sin\theta$$
$$A^\mu = -W_3^\mu \sin\theta + W'^\mu \cos\theta \qquad W_-^\mu = \sqrt{\tfrac{1}{2}}(W_1^\mu + iW_2^\mu) , \qquad (39)$$

we define

* We have been able to choose real values for both $\langle\varphi'\rangle$ and $\langle\varphi''\rangle$. Using expression (34) of the Lagrangian density we can show that time reversal is a symmetry of the broken theory, contrary to models such as T.D. Lee's, where T invariance is spontaneously broken [18].

$$\lambda_z = \lambda_3\cos\theta + \lambda'\sin\theta ,$$
$$\lambda_\gamma = -\lambda_3\sin\theta + \lambda'\cos\theta , \qquad \lambda_- = \sqrt{\tfrac{1}{2}}(\lambda_1 + i\lambda_2) , \qquad (40)$$

and similarly for D_z, D_γ and D_-.

Among the auxiliary fields D, D', H', H'', f_0 and g_0, only D_γ takes a non-vanishing vacuum expectation value:

$$\langle D_\gamma\rangle = -\xi\cos\theta . \qquad (41)$$

If ξ, g and g' are non zero, supergauge invariance is spontaneously broken. Under a gauge transformation of infinitesimal parameter α, the only term proportional to the vacuum expectation values in the variations of the fields is:

$$\delta\lambda_\gamma = -\xi\cos\theta\,\gamma_5\alpha + \ldots \qquad (42)$$

λ_γ is the massless Goldstone spinor arising from the spontaneous breakdown of supergauge invariance [2]; it is associated with the photon in a gauge vector multiplet.

We note that spontaneous breaking of both supergauge- and gauge-invariance (required for a theory of weak and electromagnetic interactions) occurs for all values of g, g', h, s and ξ, provided the symmetry group of the model is not enlarged by the choice of vanishing parameters.

5.3. Physical particles

5.3.1. Vectors

A_μ, Z_μ and $W_\mu^\pm$ have been defined in (39). A_μ is the massless photon field. The masses of the heavy vector bosons are at lowest order:

$$m_Z = \tfrac{1}{2}\sqrt{(g^2+g'^2)(v'^2+v''^2)}, \qquad m_W = \tfrac{1}{2}g\sqrt{v'^2+v''^2} . \qquad (43)$$

5.3.2. Spinors

Spontaneous symmetry breaking induces in the Lagrangian density the mass terms

$$\mathcal{L}_m = \sqrt{\tfrac{1}{2}}ig\,[(\bar\psi_{-R}v' + \bar\psi_{-L}v'')\lambda_- + \bar\lambda_-(\psi_{-R}v' + \psi_{-L}v'')]$$
$$+\tfrac{1}{2}i\sqrt{g^2+g'^2}\,[(\bar\psi_{0R}v' + \bar\psi_{0L}v'')\lambda_z + \bar\lambda_z(\psi_{0R}v' + \psi_{0L}v'')]$$
$$+\tfrac{1}{2}h\,[(\bar\psi_{0R}v'' + \bar\psi_{0L}v') \quad p - \bar p\ (\psi_{0R}v'' + \psi_{0L}v')] . \qquad (44)$$

The physical fields are

$$e_- = -\lambda_{-L} + \psi_{-R} ,$$
$$E_- = \psi_{-L} - \lambda_{-R} ,$$
$$e_0 = ip_L + (\psi_0\cos\delta + \psi_0^*\sin\delta)_R ,$$
$$E_0 = (\psi_0\cos\delta - \psi_0^*\sin\delta)_L - \lambda_{zR} ,$$
$$\nu_L = \lambda_{\gamma L} , \qquad (45)$$

where the mixing angle δ is defined by:

$$\operatorname{tg}\delta = \frac{v'}{v''} \tag{46}$$

and vanishes with s^*.

The masses are at lowest order:

$$\begin{aligned} m_{e_-} &= gv'/\sqrt{2}\,, \\ m_{E_-} &= gv''/\sqrt{2}\,, \\ m_{e_0} &= \tfrac{1}{2}h\sqrt{v'^2+v''^2}\,, \\ m_{E_0} &= \tfrac{1}{2}\sqrt{(g^2+g'^2)(v'^2+v''^2)}\,, \\ m_{\nu} &= 0\,, \end{aligned} \tag{47}$$

the last one vanishing at all orders.

The use of Dirac spinors $e_-, E_-, e_0, E_0, \nu_L$ in formula (45) is justified by consideration of R-invariance: since R leaves φ' and φ'' invariant it is still a symmetry for the broken theory. Under an R-transformation, W_μ, W'_μ, φ' and φ'' are invariant, λ, λ' and p are multiplied by $e^{\gamma_5\alpha}$, ψ by $e^{-\gamma_5\alpha}$ and $\omega = \sqrt{\tfrac{1}{2}}(a-ib)$ by $e^{2i\alpha}$. For the physical fields we have:

$$\begin{aligned} &R\,e_-\,R^{-1} = e^{i\alpha}e_-\,, \qquad R\,E_-\,R^{-1} = e^{-i\alpha}E_-\,, \\ &R\,e_0\,R^{-1} = e^{i\alpha}e_0\,, \qquad R\,E_0\,R^{-1} = e^{-i\alpha}E_0\,, \\ &R\,\nu_L\,R^{-1} = e^{i\alpha}\nu_L\,, \qquad R\,\omega\,R^{-1} = e^{2i\alpha}\,\omega\,. \end{aligned} \tag{48}$$

This gives a reason independent of Goldstone theorem for the vanishing of the λ_γ mass.

$\nu_L = \frac{1}{2}(1-i\gamma_5)\lambda_\gamma$ is identified with the neutrino field, and e_- with the electron field. We find the usual V–A expression of the charged currents.

E_-, E_0 and e_0 are heavy leptons. R-invariance can now be interpreted as the symmetry associated with lepton number conservation: e_-, e_0, ν_L have lepton number +1, E_- and E_0 −1, whereas ω is the field of a heavy scalar boson of lepton number +2.

Formulas (45) are easily inverted:

$$\psi = \begin{bmatrix} (E_0\cos\delta + e_0^*\sin\delta)_L + (-E_0^*\sin\delta + e_0\cos\delta)_R \\ E_{-L} + e_{-R} \end{bmatrix}.$$

* Here we choose $tg' < 0$. $\operatorname{tg}\delta = v'/v'' = m_{e_-}/m_{E_-}$ is small. If Q invariance is also demanded, s and m_{e_-} vanish.

$$-\lambda = \begin{bmatrix} E_{+L} + e_{+R} \\ (E_{0L}^* + E_{0R})\cos\theta + (\nu_L + \nu_R^*)\sin\theta \\ e_{-L} + E_{-R} \end{bmatrix},$$

$$\lambda' = -(E_{0L}^* + E_{0R})\sin\theta + (\nu_L + \nu_R^*)\cos\theta\,,$$

$$p = \frac{e_{0L} - e_{0R}^*}{i}\,. \tag{49}$$

We recall here that λ' and p are Majorana spinors, singlets of SU(2) × U(1); λ is a triplet of Majorana spinors and ψ a doublet of Dirac spinors with U(1) quantum numbers $F = 0$ and $F = 1$ respectively. In these expressions we clearly read the conservation of lepton number in vector boson exchanges.

Formula (49) depends on two mixing angles between neutrino fields, θ and δ. It is remarkable that in this supergauge theory θ is at the same time the SU(2) × U(1) mixing angle of Weinberg-type models [9], and the neutrino/heavy neutrino mixing angle of Georgi-Glashow type models [10]. Both θ and δ are determined by simple mass ratios:

$$\cos\theta = \frac{m_W}{m_Z}\,, \qquad \operatorname{tg}\delta = \frac{m_{e_-}}{m_{E_-}}\,. \tag{50}$$

In the heavy W limit, a W-exchange is equivalent to a local four-fermion interaction, for which we define the effective coupling constant:

$$\frac{G}{\sqrt{2}} = \frac{g^2}{4m_W^2}\sin^2\theta = \frac{e^2}{4m_W^2} \tag{51}$$

G, and hence m_W, can in principle be determined from $\bar{\nu}_e e$ scattering (see next subsection).

5.3.3. Scalar bosons.

The combinations:

$$\begin{aligned} &\varphi'_{02}\sin\delta + \varphi''_{02}\cos\delta\,, \\ &\varphi'_-\sin\delta + \varphi''_-\cos\delta\,, \end{aligned} \tag{52}$$

are as usual eliminated by the Higgs mechanism. The remaining combinations are denoted by

$$\begin{aligned} z &= -\varphi'_{01}\sin\delta + \varphi''_{01}\cos\delta\,, \\ \omega &= \frac{a-ib}{\sqrt{2}}\,, \\ \varphi &= \varphi'_0\cos\delta + \varphi''^*_0\sin\delta\,; \\ w_- &= \varphi'_-\cos\delta - \varphi''_-\sin\delta\,. \end{aligned} \tag{53}$$

Their zeroth order masses are:

$$m_\gamma = m_Z = m_{E_0} = \tfrac{1}{2}\sqrt{(g^2+g'^2)(v'^2+v''^2)}\,,$$

$$m_\omega = m_\varphi = m_{e_0} = \tfrac{1}{2}h\sqrt{v'^2+v''^2}\,,$$

$$m_W = m_W = \sqrt{\tfrac{1}{2}(m_{e_-}^2 + m_{E_-}^2)} = \tfrac{1}{2}g\sqrt{v'^2+v''^2}\,. \qquad (54)$$

5.4. Supergauge invariant extension of the Higgs model

For $\xi = 0$ parity is an extra-invariance of the Lagrangian density (34). Gauge invariance is broken spontaneously but supergauge, parity and R-invariance are conserved. The three-vector bosons $W^\pm$ and Z become massive, and supergauge invariance implies that they belong to three massive vector multiplets. For every massless vector multiplet acquiring a mass, a real scalar multiplet is eliminated by a "super-Higgs mechanism": the Majorana spinors of both multiplets join into a Dirac spinor; one of the scalars of the scalar multiplet is eliminated by the ordinary Higgs mechanism as the vector field acquires a mass; the other one remains a physical field.

In the present model with $\xi = 0$ (and then $v' = v'' = v = \sqrt{2}\,|s/h|$) the massive vector multiplets are a complex one $(W_-; E_-, e_-; w_-)$ and a real one $(Z; E_0; z)$ with masses $\sqrt{\frac{1}{2}}gv$ and $\sqrt{\frac{1}{2}(g^2+g'^2)}\,v$ respectively. $(\gamma; \lambda_\gamma)$ is still a massless vector multiplet associated with the unbroken U(1) gauge group. Two real scalar multiplets remain, rearranged into a massive complex one $(e_0; \omega, \varphi)$ by means of R-invariance; we call it physical super Higgs scalar multiplet; its mass is $\sqrt{\frac{1}{2}}hv$.

The model appears as the supergauge invariant extension of the spontaneously broken SU(2) × U(1) Yang-Mills theory. For vanishing g, we simply find the supergauge invariant extension of the well-known Higgs-Kibble model [8], exactly as the Wess and Zumino model was the supergauge invariant extension of quantum electrodynamics.

Introduction of supergauge invariance improves the Higgs-Kibble model on two points: the potential is always positive (at the tree approximation), and the gauge invariance is always spontaneously broken. No condition analogous to the usual $\lambda > 0$, $\mu^2 < 0$ has to be imposed.

We return now to the case $\xi \neq 0$. Supergauge invariance is spontaneously broken, but in a very soft way: among all F, G, and D components, D_γ only takes a non zero vacuum expectation value. This preserves, at least at lowest order, the equality of masses in the $(e_0; \omega, \varphi)$, $(Z; E_0; z)$ and $(\gamma; \nu_L)$ multiplets; in the $(W_-; E_-, e_-; w_-)$ multiplet the masses of e_- and E_- are split according to (54).

5.5. Consequences of the model

Besides the particle spectrum, well-defined predictions in electron physics are obtained.

5.5.1. $\bar{\nu}_e e$ and $\nu_e e$ scatterings.

ν_e does not appear in the weak neutral current coupled to the vector bozon Z. This is indeed a general property of all SU(2) × U(1) models where the neutrino and photon fields are associated in a massless vector multiplet. Both W_- and w_- exchanges contribute at lowest order to $\bar{\nu}_e e$ and $\nu_e e$ scatterings. The possible exchange of scalar bosons in weak processes may cause trouble to an eventual extension of the theory in order to incorporate the muon sector. The relevant part of the Lagrangian density is:

$$\mathcal{L} = g\sin\theta(W_-^\mu \bar{e}_{-L}\gamma_\mu \nu_L + i\sqrt{2}\cos\delta\, w_-\,\bar{e}_{-R}\,\nu_L) + \text{h.c.}\,, \qquad (55)$$

and we find:

$$\sigma_{\nu_e e} = \sigma_{\bar{\nu}_e e} = \frac{G^2 m_e E}{2\pi}\,\frac{16}{3}\,, \qquad (56)$$

where G has been defined in (51). These cross sections determine m_W, independently of the angle θ. Present data on $\bar{\nu}_e e$ scattering [14] give a lower bound ~ 60 GeV/c^2 for m_W.

5.5.2. Electron anomalous magnetic moment

The problem of fermion anomalous magnetic moment is important in supergauge theories. Ferrara and Remiddi [11] have shown that it vanishes identically in the model of ref. [4], as a consequence of supergauge invariance. The difficulty can be solved if supergauge invariance is spontaneously broken, as in ref. [5], but with the U(1) gauge group of quantum electrodynamics remaining unbroken. Both conditions are realized in the present model. The diagrams which contribute at the one loop level are shown in fig. 1.

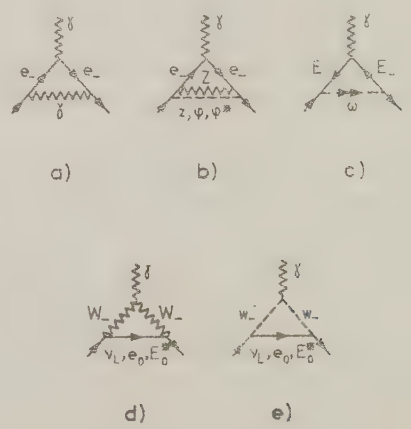

Fig. 1. One-loop diagrams associated with the electron anomalous magnetic moment. The arrows show lepton number conservation.

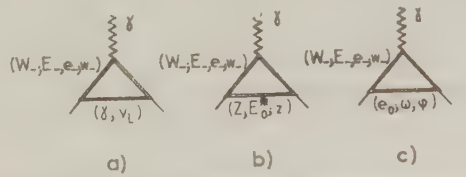

Fig. 2. One-loop "super-diagrams" associated with the electron anomalous magnetic moment.

Diagram 1a gives the well known contribution $\alpha/2\pi$ to the electron anomaly $\frac{1}{2}(g-2)$. Diagrams 1b give contributions proportional to the squared electron mass (with a possible logarithmic factor). For the other ones, whenever there is no extra damping factor $\sin\delta = m_{e_}/m_{W_}\sqrt{2}$, the initial and final electrons have the same chirality and we can retain only the chirality-conserving parts in the internal fermion propagators; thus these contributions are also proportional to the squared electron mass. So the weak contributions to the electron anomaly are negligible, provided h is not too small*.

The diagrams of fig. 1 can be associated in three diagrams shown in fig. 2, where particles of a given supermultiplet are exchanged: the photon is coupled to the charged multiplet $(W_-; E_-, e_-; w_-)$, and one of the three neutral multiplets $(\gamma; \nu_L)$, $(Z; E_0; z)$ and $(e_0; \omega, \varphi)$ is exchanged. All diagrams of fig. 1 follow immediately from superdiagrams of fig. 2 using Poincaré invariance and leptonic number conservation.

6. Conclusion

We began with building an SU(2) × U(1) gauge theory of weak and electromagnetic interactions; as in ref. [1] our leading principle was the idea that the initial theory should be as symmetric as possible, i.e. gauge- and parity-invariant; the existence of a massless neutrino was used as a phenomenological input and led to Q-invariance. Spontaneous symmetry breaking provided both the vector electromagnetic current and the usual V−A charged currents.

A connection has been established between this model, the supergauge invariant extension of quantum electrodynamics of ref. [4] and an SU(2) × U(1) supergauge theory, using the interpretation of Q-invariance in terms of superfields. But the last theory, after spontaneous symmetry breaking leads only to a massless "electron".

In order to improve it, we used a new method of spontaneous symmetry breaking which allows us to break gauge-invariance in a supergauge invariant way. We obtained the supergauge invariant extension of the Higgs-Kibble model; for every vector multiplet acquiring a mass, a real scalar multiplet is eliminated by a "super Higgs mechanism". After reduction of SU(2) × U(1) gauge invariance to U(1), the supergauge invariant theory describes the interaction of the four vector multiplets associated with γ, Z, and W_ (charged) and of a physical "super Higgs multiplet" generated by a massive Dirac spinor e_0.

Supergauge invariance was broken spontaneously as in ref. [5] but in a very soft way. The Goldstone spinor λ_γ forms with the photon γ a massless vector multiplet; it is the Majorana spinor associated with the two-component neutrino field ν_L. The masses in the multiplets remain equal, at least at lowest order, except those of the Dirac spinors e_ and E_ associated with W_: they are split on each side of the W_ mass. e_ and E_ are interpreted as the electron and heavy electron respectively. If Q-invariance were also demanded, the electron mass would vanish; the smallness of electron mass is then much more understandable than in other models with heavy leptons. R-invariance, which is still a symmetry for the broken theory, is associated with lepton number conservation. There is no trouble with the electron anomalous magnetic moment: the weak corrections are very small.

Now we know two methods for spontaneous symmetry breaking: with the method of ref. [5] supergauge invariance is broken, and gauge invariance may either be conserved, or not. We found here a supergauge-invariant way to break gauge invariance, which generalizes the Higgs mechanism without any restriction on the parameters. Applied together for an SU(2) × U(1) gauge group, the two methods give a possible model of weak and electromagnetic interactions for the electron sector.

The introduction of supergauge invariance in gauge theories has many interesting features. Spontaneous symmetry breaking is necessary. Instead of three sorts of objects in "old type" gauge theories (gauge vector fields, spinor multiplets and Higgs multiplets), we have two only: vector multiplets of generalized gauge fields ("radiation") and scalar multiplets ("matter"). Spontaneous symmetry breaking mixes the fermions of radiation and matter multiplets to give the leptons. It provides a fundamental reason for the vanishing of neutrino mass, and yields rather attractive associations of particles in multiplets: the photon with the neutrino; the charged vector boson and a scalar of the same mass with the electron and heavy electron, etc.

The model has all features desirable for a theory of weak and electromagnetic interactions, if the muon sector is ignored. Nevertheless it suggests that supergauge invariance may be of great physical interest in weak interaction theory.

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* For small h, φ exchange can be large.

References

[1] P. Fayet, Nucl. Phys. B78 (1974) 14.
[2] J. Goldstone, Nuovo Cimento 19 (1961) 154;
J. Goldstone, A. Salam and S. Weinberg, Phys. Rev. 127 (1962) 965.

[3] J. Wess and B. Zumino, Nucl. Phys. B70 (1974) 39.
[4] J. Wess and B. Zumino, Nucl. Phys. B78 (1974) 1.
[5] P. Fayet and J. Iliopoulos, Phys. Letters 51B (1974) 461.
[6] A. Salam and J. Strathdee, Nucl. Phys. B76 (1974) 477.
[7] S. Ferrara, J. Wess and B. Zumino, Phys. Letters 51B (1974) 239.
[8] B. Zumino, Lectures given at the Cargese Summer Institute, July 1972, and references therein.
[9] S. Weinberg, Phys. Rev. Letters 19 (1967) 1264;
A. Salam, Proc. of the 8th Nobel Symposium (Wiley New York, 1968).
[10] H. Georgi and S.L. Glashow, Phys. Rev. Letters 28 (1972) 1494.
[11] S. Ferrara and E. Remiddi, Preprint CERN TH 1935 (1974).
[12] P. Fayet, Proc. of the 9th Rencontre de Moriond, ed. Tran Thanh Van (1974).
[13] F.J. Hasert et al., Phys. Letters 46B (1973) 121;
D.C. Cundy, Report to the 17th Int. Conf. on high-energy physics, London (1974).
[14] H.S. Gurr, F. Reines and H.W. Sobel, Phys. Rev. Letters 28 (1972) 1406.
[15] J. Iliopoulos and B. Zumino, Nucl. Phys. B76 (1974) 310;
A. Salam and J. Strathdee, Phys. Letters 49B (1974) 465.
[16] A. Salam and J. Strathdee, Phys. Letters 51B (1974) 353.
[17] S. Ferrara and B. Zumino, Nucl. Phys. B79 (1974) 413.
[18] T.D. Lee, Phys. Rev. D8 (1973) 1226.

SPONTANEOUS SUPERSYMMETRY BREAKING WITHOUT GAUGE INVARIANCE☆

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We present a model of interacting scalar multiplets, where supersymmetry is spontaneously broken, generating a Goldstone spinor. Besides ordinary Goldstone bosons there exist scalar particles, massless at zeroth order.

Supersymmetry ‡1 transformations defined in four spacetime dimensions by Wess and Zumino [1] are generated by charges transforming, under the Lorentz group, like the components of a Majorana spinor; any linear representation contains both fermions and bosons, and it follows from the algebraic structure that all have the same mass. Therefore supersymmetry must be broken before it can be applied to the physical world. A spontaneous breaking would provide a massless-spin 1/2 Goldstone particle [2]. The simplest model one can study is the super-Φ^3 theory but no spontaneous breaking occurs: the Goldstone solution exists but is unstable [3,4].

Gauge invariant supersymmetric theories have been defined [5]. For such theories Iliopoulos and the author have shown that supersymmetry can be spontaneously broken, provided the gauge group contains an invariant abelian subgroup U(1) [6]. A Goldstone spinor is effectively obtained; gauge invariance can either be conserved or not.

Using the superfield formalism introduced in ref. [7], in a recent paper [8] we defined a group of transformations (which we called R-transformations), in order to extend the Higgs mechanism in a supersymmetric way. Then, by means of the method of ref. [6] spontaneous supersymmetry breaking was obtained. The resulting model can be taken as describing weak and electromagnetic interactions for the electron sector; the neutrino, which is the Goldstone spinor, is associated with the photon in a massless vector multiplet; R-invariance gives lepton-number conservation.

Here we use R-invariance in order to show that supersymmetry can be spontaneously broken, even in the absence of gauge invariance (this seemed impossible according to a heuristic argument given in ref. [3], which was shown recently to be invalid [9]). The method applies also for a gauge theory, and is especially useful in the case of semi-simple gauge groups such as SU(2).

In a supersymmetric field theory of interacting scalar multiplets the situation is generally the following: the potential, computed at the tree approximation, has an absolute minimum equal to zero. Eventually, one translates the A and B components of the multiplets in order to choose a "vacuum state" for which the potential vanishes. All F and G components have vanishing vacuum expectation values, and supersymmetry is conserved.

We can avoid this, by means of the R-invariance defined in ref. [8], which restricts the possible couplings of scalar multiplets. The model we present now, invariant under global SU(2) × U(1) transformations, describes the interactions of a singlet $\mathcal{S}$ and a triplet $\vec{\mathcal{T}}$ of real scalar multiplets with a doublet $\mathcal{D}$ of complex ones. We denote

$$\mathcal{D}=\begin{bmatrix}(\mathcal{D}_{u1}+i\mathcal{D}_{u2})/\sqrt{2}\\(\mathcal{D}_{d1}+i\mathcal{D}_{d2})/\sqrt{2}\end{bmatrix}, \quad \text{and call } \underline{\mathcal{S}}, \underline{\vec{\mathcal{T}}} \text{ and } \underline{\mathcal{D}}_{ki}\ (k=u,d;\ i=1,2),$$

the left-handed superfields associated with the real scalar multiplets $\mathcal{S}$, $\vec{\mathcal{T}}$ and $\mathcal{D}_{ki}$. They transform under R [8] like:

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‡1 Supersymmetry was originally called "supergauge symmetry".

$$
\begin{aligned}
R\underline{\mathcal{O}}(x,\theta)R^{-1} &= \mathrm{e}^{2\mathrm{i}\alpha}\underline{\mathcal{O}}(x,\theta\,\mathrm{e}^{-\mathrm{i}\alpha})\,,\\
R\underline{\vec{\mathcal{T}}}(x,\theta)R^{-1} &= \mathrm{e}^{2\mathrm{i}\alpha}\underline{\vec{\mathcal{T}}}(x,\theta\mathrm{e}^{-\mathrm{i}\alpha})\,,\\
R\underline{\mathcal{D}}_{ki}(x,\theta)R^{-1} &= \underline{\mathcal{D}}_{ki}(x,\theta\,\mathrm{e}^{-\mathrm{i}\alpha})\,.
\end{aligned}
\tag{1}
$$

Owing to supersymmetry, SU(2) × U(1), R- and parity invariances, the most general Lagrangian density renormalisable by power counting is:

$$
\mathcal{L}=\tfrac{1}{4}\left[\mathcal{O}\times\mathcal{O}+2\mathcal{D}^{\dagger}\times\mathcal{D}+\vec{\mathcal{T}}\times\vec{\mathcal{T}}\right]_{\text{D component}}+\left[s\mathcal{O}+\tfrac{1}{2}h'\mathcal{D}^{\dagger}\,\mathcal{D}\mathcal{O}+\tfrac{1}{2}h\mathcal{D}^{\dagger}\vec{\tau}\mathcal{D}\vec{\mathcal{T}}\right]_{\text{F component}}. \tag{2}
$$

R-invariance forbids the direct mass terms, together with the cubic self-interaction of the $\mathcal{O}$ multiplet.

Let (a,b,p), $(\boldsymbol{a},\boldsymbol{b},\boldsymbol{p})$ and (A,B,ψ) be the components of $\mathcal{O}$, $\vec{\mathcal{T}}$ and $\mathcal{D}$ respectively. We use also

$$
\begin{cases}
\varphi' = -\mathrm{i}\dfrac{A+\mathrm{i}B}{\sqrt{2}} & \begin{cases} H' = -\mathrm{i}\dfrac{F+\mathrm{i}G}{\sqrt{2}} \\ H'' = -\mathrm{i}\dfrac{F-\mathrm{i}G}{\sqrt{2}} \end{cases}\\
\varphi'' = -\mathrm{i}\dfrac{A-\mathrm{i}B}{\sqrt{2}}
\end{cases}
\tag{3}
$$

The Lagrangian density reads:

$$
\begin{aligned}
\mathcal{L} &= \tfrac{1}{2}\left[f^2+g^2-\partial_\mu a\partial^\mu a-\partial_\mu b\partial^\mu b-\mathrm{i}\bar{p}\partial\!\!\!/ p\right]+sf+\tfrac{1}{2}\left[\boldsymbol{f}^2+\boldsymbol{g}^2-\partial_\mu\boldsymbol{a}\quad\partial^\mu\boldsymbol{a}-\partial_\mu\boldsymbol{b}\,\partial^\mu\boldsymbol{b}-\mathrm{i}\bar{\boldsymbol{p}}\partial\!\!\!/\boldsymbol{p}\right]\\
&+H'^{\dagger}H'+H''^{\dagger}H''-\partial_\mu\varphi'^{\dagger}\partial^\mu\varphi'-\partial_\mu\varphi''^{\dagger}\partial^\mu\varphi''-\mathrm{i}\bar{\psi}\partial\!\!\!/\psi+\tfrac{1}{2}h'\{(f+\mathrm{i}g)\varphi'^{\dagger}\varphi''+(f-\mathrm{i}g)\varphi''^{\dagger}\varphi'\\
&+(a+\mathrm{i}b)(H'^{\dagger}\varphi'+\varphi''^{\dagger}H'')+(a-\mathrm{i}b)(H''^{\dagger}\varphi''+\varphi'^{\dagger}H')-\mathrm{i}\bar{\psi}(a-\gamma_5 b)\psi\\
&+\sqrt{2}\left[(\bar{\psi}_{\mathrm{R}}\varphi''+\bar{\psi}_{\mathrm{L}}\varphi')p-\bar{p}(\varphi''^{\dagger}\psi_{\mathrm{R}}+\varphi'^{\dagger}\psi_{\mathrm{L}})\right]\}+\tfrac{1}{2}h\{f+\mathrm{i}g)\varphi'^{\dagger}\tau\varphi''+(f-\mathrm{i}g)\varphi''^{\dagger}\tau\varphi'\\
&+(\boldsymbol{a}+\mathrm{i}\boldsymbol{b})(H'^{\dagger}\tau\varphi'+\varphi''^{\dagger}\tau H'')+(\boldsymbol{a}-\mathrm{i}\boldsymbol{b})(H''^{\dagger}\tau\varphi''+\varphi'^{\dagger}\tau H')-\mathrm{i}\bar{\psi}(a-\gamma_5 b)\tau\psi\\
&+\sqrt{2}\left[(\bar{\psi}_{\mathrm{R}}\tau\varphi''+\bar{\psi}_{\mathrm{L}}\tau\varphi')\boldsymbol{p}-\bar{\boldsymbol{p}}(\varphi''^{\dagger}\tau\psi_{\mathrm{R}}+\varphi'^{\dagger}\tau\psi_{\mathrm{L}})\right]\}\,.
\end{aligned}
\tag{4}
$$

The auxiliary fields are expressed in terms of the physical ones:

$$
\begin{cases}
f+\mathrm{i}g+s+h'\varphi''^{\dagger}\varphi'=0\\
\boldsymbol{f}+\mathrm{i}\boldsymbol{g}+h\varphi''^{\dagger}\tau\varphi'=0\\
H'+\tfrac{1}{2}\left[h'(\boldsymbol{a}+\mathrm{i}\boldsymbol{b})+h(\boldsymbol{a}+\mathrm{i}\boldsymbol{b})\tau\right]\varphi'=0\\
H''+\tfrac{1}{2}\left[h'(\boldsymbol{a}-\mathrm{i}\boldsymbol{b})+h(\boldsymbol{a}-\mathrm{i}\boldsymbol{b})\tau\right]\varphi''=0\,.
\end{cases}
\tag{5}
$$

If s and h do not vanish, we cannot have both $f+\mathrm{i}g=0$ and $\boldsymbol{f}+\mathrm{i}\boldsymbol{g}=0$. The potential, which equals

$$
V_0=\tfrac{1}{2}|f+\mathrm{i}g|^2+\tfrac{1}{2}|\boldsymbol{f}+\mathrm{i}\boldsymbol{g}|^2+H'^{\dagger}H'+H''^{\dagger}H''\,, \tag{6}
$$

at the tree approximation, has an absolute minimum strictly positive. It follows that supersymmetry is spontaneously broken, as we shall verify later.

Now we search for the minimum of V_0. For these points the global SU(2) × U(1) invariance is spontaneously broken down to a U(1) subgroup, associated with a conserved quantum number we call "charge". Using the SU(2) × U(1) freedom we can choose:

$$
\langle\varphi'\rangle=\begin{bmatrix} v'/\sqrt{2} \\ 0 \end{bmatrix},\qquad \langle\varphi''\rangle=\begin{bmatrix} v''/\sqrt{2} \\ 0 \end{bmatrix},
$$

with v' and v'' real and we will have three Goldstone bosons.

Still the minimum of V_0 is not uniquely defined. We have $v'v'' = -2h's/(h^2 + h'^2)$ but do not know v' and v'' separately. Moreover when V_0 takes its minimum value, $H' = H'' = 0$, which is not enough to fix the vacuum expectation values for all $a, b, \boldsymbol{a}$ and $\boldsymbol{b}$ fields.

Therefore one must compute higher order corrections to the potential in order to choose between all vacuum states that appear possible at the tree approximation. It follows that, besides the three massless Goldstone bosons there exist scalar particles massless at zeroth order [10]. Contrary to the case of pseudo-Goldstone bosons [11] (that appear when V_0 has a symmetry group larger than V), the zeroth order mass spectrum of scalar particles here depends on the vacuum state selected by higher order corrections.

We define the combinations

$$\begin{aligned} \mathcal{S}_z &= \cos\theta\ \mathcal{T}_3 + \sin\theta\ \mathcal{Y} \\ \mathcal{S}_\gamma &= -\sin\theta\ \mathcal{T}_3 + \cos\theta\ \mathcal{Y} \end{aligned} \tag{7}$$

with $\tan\theta = h'/h$. Whatever vacuum state is chosen, all vacuum expectation values for f, g, f, g, H' and H'' vanish except for f_γ.

If we perform a supersymmetry transformation of infinitesimal parameter α, we find that

$$p_\gamma = -\sin\theta\, p_3 + \cos\theta\, p\ ,$$

is the Goldstone spinor, whose mass vanishes at all orders. To prove that supersymmetry breaking has observable consequences, we remark that the two "charged"[‡2] Dirac spinors are massive, whereas among the four "charged" spinless fields associated with them one is a massless Goldstone field.

We cannot say more without computing higher order corrections. Nevertheless, in order to illustrate the above considerations we shall diagonalize the zeroth order mass matrix, with the extra assumption (which may be not verified) that the vacuum state is R-invariant. With $\tan\delta = v'/v''$ we find:

$$\begin{cases} m\left[\Phi'_- = -\mathrm{i}\dfrac{a_- + \mathrm{i}b_-}{\sqrt{2}}\right] & = hv'/\sqrt{2}\ , \\ m\left[\Phi''_- = -\mathrm{i}\dfrac{a_- - \mathrm{i}b_-}{\sqrt{2}}\right] & = hv''/\sqrt{2}\ , \end{cases}$$

$$\begin{cases} m\,[\omega_- = \cos\delta\ \varphi'_- - \sin\delta\ \varphi''_- & = h\sqrt{(v'^2 + v''^2)/2}\ , \\ m\,[\tilde{\omega}_- = \sin\delta\ \varphi'_- + \cos\delta\ \varphi''_- & = 0\ , \end{cases} \tag{8}$$

$$\begin{cases} m\,[e_- = \mathrm{i}p_{-\mathrm{L}} + \psi_{-\mathrm{R}}] & = hv''/\sqrt{2}\ , \\ m\,[E_- = \psi_{-\mathrm{L}} + \mathrm{i}p_{-\mathrm{R}}] & = hv'/\sqrt{2}\ . \end{cases}$$

On the other hand:

$$\left\{\omega_z = \frac{a_z - \mathrm{i}b_z}{\sqrt{2}};\qquad \varphi = \cos\delta\ \varphi'_0 + \sin\delta\ \varphi''^*_0;\qquad e_0 = \mathrm{i}p_{z\mathrm{L}} + (\cos\delta\ \psi_0 + \sin\delta\,\psi^*_0)_\mathrm{R}\right\},$$

have zeroth order mass $\frac{1}{2}\sqrt{(h^2 + h'^2)(v'^2 + v''^2)}$.

$$\left\{\omega_\gamma = \frac{a_\gamma - \mathrm{i}b_\gamma}{\sqrt{2}};\qquad E_{0\mathrm{R}} = \mathrm{i}p_{\gamma\mathrm{R}}\right\},$$

[‡2] "Charge" has been defined previously as the conserved quantum number associated with the U(1) invariance remaining after spontaneous breaking of the global SU(2) × U(1) symmetry group.

together with

$$\{z = -\sin\delta\,\varphi'_{01} + \cos\delta\,\varphi''_{01}\,; \qquad \tilde{z} = \sin\delta\,\varphi'_{02} + \cos\delta\,\varphi''_{02}\,; \qquad E_{0L} = (\cos\delta\,\psi_0 - \sin\delta\,\psi_0^*)_L\}\,,$$

have vanishing zeroth order mass.

p_γ is the Goldstone spinor; by means of R-invariance we find that E_{0L} also is massless at all orders. $\tilde{w}_-$ and $\tilde{z}$ are the Goldstone bosons; ω_γ and z are massless at zeroth order.

The point for which both R-invariance and parity are conserved is stationary for the potential V calculated to all orders. Possibly it is the true minimum of V, in which case the zeroth order mass spectrum is completely determined (put $v' = v''$ in the above formulas).

All that can be extended very easily if one (or both) of the invariances SU(2) and U(1) is realized locally. The usual Higgs–Kibble mechanism is then operating to eliminate Goldstone bosons.

To sum up, we have obtained the following new results (without any restriction on the parameters of the model):

– Supersymmetry can be spontaneously broken in the absence of gauge group, or if the gauge group is semi-simple. Possibly parity is conserved in the breaking.

– Scalar particles appear, whose masses vanish at zeroth order and are calculable at higher orders[‡3].

I am very grateful to Professor J. Wess and the members of the Institut für Theoretische Physik at Karlsruhe for their kind hospitality during December 1974, and for many interesting discussions. I wish also to thank Professor J. Iliopoulos for useful comments.

[‡3] After this work was completed we received a paper from O'Raifeartaigh: "On the question of global stability for the spontaneous breakdown of chiral scalar supersymmetry". In his paper the Lagrangian density does not contain all terms compatible with renormalization and symmetries. Here on the contrary we used R-invariance in order to restrict the possible Lagrangian density, and obtained supersymmetry breaking for all (non vanishing) values of the parameters.

References

[1] J. Wess and B. Zumino, Nucl. Phys. B70 (1974) 39.
[2] J. Goldstone, Novo Cimento 19 (1961) 154;
J. Goldstone, A. Salam and S. Weinberg, Phys. Rev. 127 (1962) 965.
[3] J. Iliopoulos and B. Zumino, Nucl. Phys. B76 (1974) 310.
[4] A. Salam and J. Strathdee, Phys. Letters 49B (1974) 465.
[5] J. Wess and B. Zumino, Nucl. Phys. B78 (1974) 1.
[6] P. Fayet and J. Iliopoulos, Phys. Letters 51B (1974) 461.
[7] A. Salam and J. Strathdee, Nucl. Phys. B76 (1974) 477.
[8] P. Fayet, preprint PTENS 74/7, to be published in Nucl. Phys. B.
[9] B. Zumino, preprint CERN TH. 1942.
[10] S. Weinberg, report to the 2nd Int. Conf. on Elementary particles at AIX-EN-PROVENCE, 1973, § 7, and references therein.
[11] S. Weinberg, Phys. Rev. Lett. 29 (1972) 1698.

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Higgs Model and Supersymmetry (*).

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Summary. — The Higgs model can be made supersymmetric, provided a Dirac spinor is added, and the quartic interaction term in the Lagrangian density is $-(e^2/2)(\varphi^\dagger\varphi)^2$, where e is the gauge coupling constant. The model describes a vector, a Dirac spinor and a real scalar; it depends only on two arbitrary parameters, m and e. As a gauge-invariant model, it can be studied in a Wess-Zumino gauge, or in a supersymmetric gauge. It can also be obtained as a theory of a self-interacting vector multiplet. Finally, it is suggested that a scalar particle may be associated with the photon and the two neutrinos in the theory of weak and electromagnetic interactions.

1. – Introduction.

Supersymmetry [1] implies strong constraints on field theory models, which one may use to reduce the number of arbitrary parameters. Here we want to embed the Higgs model [2], a basic ingredient of spontaneously broken gauge theories, in a supersymmetric one. If supersymmetry is conserved, the model describes, at least, one massive vector multiplet including a vector, a Dirac spinor and a real scalar, of equal masses.

A solution was proposed in sect. **5** of ref. [3]: after spontaneous breaking of gauge invariance, this model described the supersymmetric-parity–conserv-

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(1) J. Wess and B. Zumino: *Nucl. Phys.*, **70** B, 39 (1974).

(2) P. W. Higgs: *Phys. Rev. Lett.*, **13**, 508 (1964); F. Englert and R. Brout: *Phys. Rev. Lett.*, **13**, 321 (1964).

(3) P. Fayet: *Nucl. Phys.*, **90** B, 104 (1975).

ing interaction of a massive vector multiplet with a complex scalar one (see appendix). With $SU_2 \times U_1$ as a gauge group and after spontaneous supersymmetry breaking, the method provided a model of weak and electromagnetic interactions for the electron sector.

We present here the smallest supersymmetric extension of the Higgs model: it involves only one massive vector multiplet alone. Let $\mathscr{V}$ be a massless vector multiplet of « U_1 generalized gauge fields » [4], including a vector V_μ and a Majorana spinor λ; $\mathscr{S}$ is a left-handed scalar multiplet, including a left-handed Dirac spinor ψ_L and a complex scalar φ. The model describes the gauge-invariant interaction of $\mathscr{V}$ with $\mathscr{S}$.

There are two different situations: if supersymmetry is conserved, gauge invariance is spontaneously broken, and *vice versa*. In the first case the physical fields are a vector, a Dirac spinor and a real scalar, with the same mass: the model describes the supersymmetric self-interaction of a massive vector multiplet. Instead of three arbitrary parameters in the Higgs model, the present one depends only on two: the mass and the coupling constant. We present in sect. **2** nontechnical results relative to both situations of spontaneous symmetry breaking.

In sect. **3** we study the model in two different gauges: the Wess-Zumino (WZ) gauge, which does not preserve supersymmetry, and a supersymmetric gauge in which the scalar multiplet is eliminated, the vector multiplet remaining alone.

In sect. **4** the same model is obtained as follows: we study a class of supersymmetric self-interactions for a vector multiplet and require that interaction terms quartic with respect to the spinor fields eliminate. The Lagrangian density is identified with that found previously in the supersymmetric gauge. When supersymmetry is conserved, but not gauge invariance, we recover directly the Lagrangian density of a spontaneously broken gauge theory, after elimination of the Goldstone boson.

In sect. **5** we construct explicitly the generalized gauge transformation connecting the supersymmetric to the WZ gauges.

Finally, in sect. **6**, we exhibit two conserved quantum numbers when gauge invariance is preserved; in the limit of vanishing coupling constant, it is suggested that the fields can be used to represent the photon, the two neutrinos and a weak-interacting scalar particle ω.

2. – Summary of nontechnical results.

In this section we present results which can be understood without technical knowledge about supersymmetry. We first review the Higgs model.

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(4) J. WESS and B. ZUMINO: *Nucl. Phys.*, **78** B, 1 (1974).

It describes the U_1-invariant interaction of a gauge field V_μ with a complex scalar field φ. Let the covariant derivative be defined by

$$iD_\mu = i\partial_\mu - eV_\mu , \tag{1}$$

where we can assume $e > 0$. The Lagrangian density, which depends on three arbitrary parameters e, h and μ^2, is

$$\mathscr{L} = -\tfrac{1}{4} V_{\mu\nu} V^{\mu\nu} - D_\mu \varphi^\dagger D^\mu \varphi - \mu^2 \varphi^\dagger \varphi - h(\varphi^\dagger \varphi)^2 . \tag{2}$$

If $h > 0$, $\mu^2 < 0$, gauge invariance is spontaneously broken, and we are left with a vector field and a real scalar one, both massive.

Supersymmetry associates a Majorana spinor λ with the vector field V_μ; with the complex scalar φ a left-handed (*) Dirac spinor ψ_L. (V_μ, λ) is a « massless vector multiplet of generalized gauge fields »; it interacts in a U_1-invariant way with the « left-handed scalar multiplet » (ψ_L, φ). The Lagrangian density is

$$\mathscr{L} = -\frac{1}{4} V_{\mu\nu} V^{\mu\nu} - \frac{i}{2} \bar{\lambda} \partial \lambda - i \bar{\psi}_L \hat{D} \psi_L - D_\mu \varphi^\dagger D^\mu \varphi + \\ + ie\sqrt{2} [\bar{\psi}_L \lambda \varphi + \varphi^\dagger \bar{\lambda} \psi_L] - \frac{1}{2} [\xi + e\varphi^\dagger \varphi]^2 . \tag{3}$$

If $\xi > 0$, the potential is minimum for vanishing φ; gauge invariance is conserved, but supersymmetry is not. The model describes a vector V_μ, two left-handed Dirac spinors ψ_L and λ_L, all massless, and a complex scalar φ, of mass $\sqrt{\xi e}$. λ_L is the Goldstone spinor arising from spontaneous supersymmetry breaking.

If $\xi < 0$, the potential has a vanishing minimum for nonvanishing φ: supersymmetry is conserved, but gauge invariance is not.

Let us write

$$\begin{cases} \varphi = -i \dfrac{A - iB}{\sqrt{2}} , \\ \psi_L + \lambda_R = E . \end{cases} \tag{4}$$

We can choose

$$\begin{cases} \langle A \rangle = 0 , \\ \langle B \rangle = v , \end{cases} \tag{5}$$

(*) The subscripts L and R are relative to the $(1 - i\gamma_5)/2$ and $(1 + i\gamma_5)/2$ parts of the spinor fields, respectively.

where v is a positive real number fixed by

$$\xi + \tfrac{1}{2} e v^2 = 0 \,. \tag{6}$$

We translate the B-field. A is the Goldstone boson, eliminated by the usual Higgs mechanism as the vector boson becomes massive. The physical fields are the vector W_μ, the Dirac spinor E and the real scalar B_t. They all have the same mass $m = ev$ and form a « massive vector multiplet ». Its supersymmetric self-interaction is described by the (translated) Lagrangian density

$$\begin{aligned} \mathscr{L} = &-\frac{1}{4} W_{\mu\nu} W^{\mu\nu} - i\bar{E}\partial\!\!\!/ E - \frac{1}{2}\,\partial_\mu B_t \partial^\mu B_t - \\ &-\frac{1}{2}(m + eB_t)^2 W_\mu W^\mu - \frac{1}{2}\left(mB_t + \frac{eB_t^2}{2}\right)^2 - i(m + eB_t)\bar{E}E + e\bar{E}_L \hat{W} E_L \,. \end{aligned} \tag{7}$$

Let us compare the results with those of the Higgs model. The part relative to the vector W_μ and the real scalar B_t in (7) is identical to that obtained from (2), when the extra relation $h = e^2/2$ is required. Adding the Dirac spinor E we made the Higgs model supersymmetric; it depends now on two parameters instead of three: the mass m and the coupling constant e.

If gauge invariance is spontaneously broken, supersymmetry is not, and the potential has a vanishing vacuum expectation value: there is no vacuum energy, and no cosmological term is induced in an external gravitational field ([5]).

An unpleasant feature of the model is that renormalizability is spoilt by the anomaly associated with the axial vector current. This problem can be solved by adding a Dirac spinor and two complex scalar fields; the new model still depends only on the two parameters m and e ([6]).

3. – Gauge-invariant interaction of a massless vector multiplet with a left-handed scalar one.

We use (for convenience only) the superfield formalism ([7]) to express the Lagrangian density. θ is an element of a Grassmann algebra of anticommuting two-component complex spinors, and $\bar{\theta}$ its conjugate. The multiplets $\mathscr{V}$ and $\mathscr{S}$ are described respectively by the real superfield V and the left-handed superfield S. $\mathscr{L}_0$ is the free Lagrangian density for the massless vector multiplet,

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([5]) B. Zumino: *Nucl. Phys.*, **89** B, 535 (1975).
([6]) P. Fayet: preprint PTENS 75/8.
([7]) A. Salam and J. Strathdee: *Nucl. Phys.*, **76** B, 477 (1974); S. Ferrara, J. Wess and B. Zumino: *Phys. Lett.*, **51** B, 239 (1974).

to which we add the term ξD as in ref. ([8]). The most general Lagrangian density compatible with supersymmetry and gauge invariance of the action reads

$$\mathscr{L} = \mathscr{L}_0 + \xi D + [\mathcal{S}^* \exp[2eV]\mathcal{S}]_{D\text{-component}} . \tag{8}$$

It is an infinite power series in the coupling constant e. In the Wess-Zumino gauge, for which supersymmetry is no longer manifest, V_μ, λ and D are the only components of $\mathscr{V}$ which do not vanish. The Lagrangian density is quadratic in e and reads (see, for example, formula (34) of ref. ([3]))

$$\mathscr{L} = \left[-\frac{1}{4}V_{\mu\nu}V^{\mu\nu} - \frac{i}{2}\bar{\lambda}\partial\!\!\!/\lambda + \frac{D^2}{2}\right] + \xi D - \tag{9}$$
$$- i\bar{\psi}_L \hat{D}\psi_L - D_\mu\varphi^\dagger D^\mu\varphi + H^\dagger H + ie\sqrt{2}[\bar{\psi}_L\lambda\varphi + \varphi^\dagger\bar{\lambda}\psi_L] + eD\varphi^\dagger\varphi .$$

D and $H = -i((F + iG)/\sqrt{2})$ are auxiliary fields. Their elimination gives expression (3) for the Lagrangian density; all results in sect. **2** are derived immediately.

Instead of the Wess-Zumino gauge, we can choose one in which

$$\mathcal{S} = -\frac{iv}{2} . \tag{10}$$

When $\xi < 0$ (spontaneously broken gauge invariance), we shall choose for v the vacuum expectation value of the B-field. Condition (10) is invariant under a supersymmetry transformation. In this supersymmetric gauge, the vector multiplet is denoted by $\mathscr{V}'$, the corresponding superfield by W. The Lagrangian density reads

$$\mathscr{L} = \mathscr{L}_0 + \left[\xi W + \frac{v^2}{4}\exp[2eW]\right]_D . \tag{11}$$

For $\xi < 0$ we develop this expression in a power series; the linear term vanishes, since

$$\xi + \tfrac{1}{2}ev^2 = 0 . \tag{12}$$

The expansion of $\mathscr{L}$ up to second-order terms is the Lagrangian density for a free vector multiplet of mass $m = ev$:

$$\mathscr{L} = \mathscr{L}_0 + \tfrac{1}{2}m^2[W^2]_D . \tag{13}$$

([8]) P. FAYET and J. ILIOPOULOS: *Phys. Lett.*, **51** B, 461 (1974).

The Wess-Zumino gauge is related to the supersymmetric gauge by means of the generalized gauge transformation

$$(14)\qquad \begin{cases} S \to S \exp[-2ieA] = -\dfrac{iv}{2}, \\ V \to V + i(A - A^\dagger) = W. \end{cases}$$

This connection will be studied in detail in sect. **5**. Note that the idea of supersymmetric gauge (as defined here) can be easily extended to non-Abelian gauge models, provided there exist scalar multiplets in convenient representations of the gauge group.

4. – Self-interacting vector multiplet.

4·1. *The free vector multiplet.* – We recall the main results to define notations. A vector multiplet $\mathscr{W}$ includes a vector W_μ, two Majorana spinors λ and χ and real scalar fields C, M, N and D. The Lagrangian density for a free massless vector multiplet is

$$(15)\qquad \mathscr{L}_0 = -\frac{1}{4} W_{\mu\nu} W^{\mu\nu} - \frac{i}{2} \bar{\lambda} \partial \lambda + \frac{D^2}{2}.$$

All fields except W_μ and λ are auxiliary ones.

Formula (5) of ref. [4] defines the vector multiplet $\mathscr{W}^2$. Under a supersymmetry transformation the D-component of any vector multiplet is changed by a total derivative, and can be used to construct a Lagrangian density. For a free massive vector multiplet we have

$$(16)\qquad \mathscr{L} = \mathscr{L}_0 + \frac{m^2}{2} [\mathscr{W}^2]_D = -\frac{1}{4} W_{\mu\nu} W^{\mu\nu} - \frac{i}{2} \bar{\lambda} \partial \lambda + \frac{D^2}{2} +$$
$$+ \frac{m^2}{2} (2CD - \partial_\mu C \partial^\mu C - W_\mu W^\mu + M^2 + N^2 - 2i\bar{\chi}\lambda - i\bar{\chi}\partial\chi).$$

We eliminate the auxiliary fields D, M, N, and define

$$(17)\qquad \begin{cases} B_t = mC, \\ E = m\chi_L + \lambda_R. \end{cases}$$

Then

$$(18)\qquad \mathscr{L} = -\frac{1}{4} W_{\mu\nu} W^{\mu\nu} - i\bar{E}\partial E - \frac{1}{2} \partial_\mu B_t \partial^\mu B_t - \frac{m^2}{2} W_\mu W^\mu - im\bar{E}E - \frac{m^2}{2} B_t^2.$$

The massive vector multiplet includes, as physical fields, the vector W_μ, the Dirac spinor E and the real scalar B_i, of the same mass m.

4·2. *The self-interaction $f(\mathscr{W})$.* – We are interested in the mass + self-interaction of the vector multiplet, described by the Lagrangian density

$$\mathscr{L} = \mathscr{L}_0 + [f(\mathscr{W})]_D \,, \tag{19}$$

where the real function f and its derivatives can be expanded as a power series. After some computation using in ref. (4) formula (5) and those of the appendix, we find

$$\begin{aligned} \mathscr{L} = {} & \left(-\frac{1}{4} W_{\mu\nu} W^{\mu\nu} - \frac{i}{2} \bar{\lambda} \partial\!\!\!/ \lambda + \frac{D^2}{2}\right) + f^{(1)}(C) D + \\ & + \frac{1}{2} f^{(2)}(C)(-\partial_\mu C \partial^\mu C - W_\mu W^\mu + M^2 + N^2 - 2i\bar{\chi}\lambda - i\bar{\chi}\partial\!\!\!/\chi) - \\ & - \frac{i}{4} f^{(3)}(C) \bar{\chi}(M\gamma_5 + N - \gamma_5 \hat{W})\chi - \frac{1}{16} f^{(4)}(C)(\bar{\chi}\chi)^2 \,. \end{aligned} \tag{20}$$

We suppose $f^{(2)}(C)$ positive, so that the Hamiltonian density is lower-bounded. The auxiliary fields M, N and D are eliminated by using the equations of motion

$$\left\{ \begin{aligned} & D + f^{(1)}(C) = 0 \,, \\ & M f^{(2)}(C) - \frac{i}{4} f^{(3)}(C) \bar{\chi}\gamma_5 \chi = 0 \,, \\ & N f^{(2)}(C) - \frac{i}{4} f^{(3)}(C) \bar{\chi}\chi = 0 \,, \end{aligned} \right. \tag{21}$$

and the Lagrangian density becomes

$$\begin{aligned} \mathscr{L} = {} & -\frac{1}{4} W_{\mu\nu} W^{\mu\nu} - \frac{i}{2} \bar{\lambda} \partial\!\!\!/ \lambda - \frac{1}{2} (f^{(1)}(C))^2 + \\ & + \frac{1}{2} f^{(2)}(C)(-\partial_\mu C \partial^\mu C - W_\mu W^\mu - 2i\bar{\chi}\lambda - i\bar{\chi}\partial\!\!\!/\chi) + \\ & + \frac{i}{4} f^{(3)}(C) \bar{\chi}\gamma_5 \hat{W}\chi + \left\{ \frac{[f^{(3)}(C)]^2}{f^{(2)}(C)} - f^{(4)}(C) \right\} \frac{(\bar{\chi}\chi)^2}{16} \,. \end{aligned} \tag{22}$$

The last term vanishes if f satisfies the differential equation

$$(f^{(3)})^2 = f^{(2)} \times f^{(4)} \,. \tag{23}$$

The second derivative of f is either a constant or an exponential function. In the first case we have

$$\mathscr{L} = \mathscr{L}_0 + \left[\xi \mathscr{W} + \frac{m^2}{2} \mathscr{W}^2\right]_D . \tag{24}$$

The linear term can be eliminated by means of a translation on C. This Lagrangian density describes a free vector multiplet of mass m.

Except this trivial case, the general solution is

$$\mathscr{L} = \mathscr{L}_0 + \left[\xi \mathscr{W} + \frac{v^2}{4} \exp[2e\mathscr{W}]\right]_D . \tag{25}$$

We choose $e > 0$ without loss of generality. ξ and e are the only parameters of the model: v has no physical meaning since a translation on C can still be performed.

As announced in the introduction, we find again expression (11) for the Lagrangian density. Studying self-interactions for a vector multiplet, we have recovered the gauge-invariant model of sect. **3**, when the supersymmetric gauge is chosen.

The complete expression for the Lagrangian density is

$$\begin{aligned} \mathscr{L} = {} & \left(-\frac{1}{4} W_{\mu\nu} W^{\mu\nu} - \frac{i}{2} \bar{\lambda} \partial \lambda + \frac{D^2}{2}\right) + \left(\xi + \frac{e v^2}{2} \exp[2eC]\right) D - \\ & - \frac{1}{2} e^2 v^2 \exp[2eC] (\partial_\mu C \partial^\mu C + W_\mu W^\mu + 2 i \bar{\chi} \lambda + i \bar{\chi} \partial \chi) + \\ & + i \frac{e^3 v^2}{2} \exp[2eC] \bar{\chi} \gamma_5 \hat{W} \chi + \frac{e^2 v^2}{2} \exp[2eC] \left[\left(M - \frac{ie}{2} \bar{\chi} \gamma_5 \chi\right)^2 + \left(N - \frac{ie}{2} \bar{\chi} \chi\right)^2\right] . \end{aligned} \tag{26}$$

Terms quartic with respect to spinor fields clearly disappear when auxiliary fields are eliminated.

4·3. *The two cases of spontaneous symmetry breaking.* – The potential of the scalar field is

$$\mathscr{U}(C) = \frac{1}{2} \left(\xi + \frac{e v^2}{2} \exp[2eC]\right)^2 . \tag{27}$$

If $\xi > 0$, the potential has a nonvanishing minimum: the auxiliary field D has the vacuum expectation value $-\xi$. Supersymmetry is spontaneously broken and λ is the Goldstone spinor. Since $\langle \exp[2eC] \rangle = 0$, new fields have to be defined: we shall do this in sect. **5**.

If $\xi < 0$ the potential has a vanishing minimum and supersymmetry is conserved. Since the model does not depend on the parameter v, we choose

the positive solution of

$$\xi + \tfrac{1}{2} e v^2 = 0 . \tag{28}$$

No translation on C has to be performed. Defining the new fields (*)

$$\begin{cases} B = v \exp [eC] , \\ \psi = ev \exp [eC] \chi , \end{cases} \tag{29}$$

we find

$$\mathscr{L} = -\frac{1}{4} W_{\mu\nu} W^{\mu\nu} - \frac{i}{2} \bar{\lambda} \partial \lambda - \frac{1}{2} \left(\xi + \frac{eB^2}{2} \right)^2 - \tag{30}$$

$$- \frac{1}{2} \partial_\mu B \partial^\mu B - \frac{e^2}{2} W_\mu W^\mu B^2 - ieB \bar{\psi} \lambda - \frac{i}{2} \bar{\psi} \partial \psi + \frac{ie}{2} \bar{\psi} \gamma_5 \hat{W} \psi .$$

This formula is the same as (3), when the gauge is chosen in order to eliminate the A-field. Since C has a vanishing expectation value, we still have to translate the B-field. With

$$\begin{cases} B = v + B_t , \\ \psi_L + \lambda_R = E , \end{cases} \tag{31}$$

we recover the Lagrangian density (7) of the spontaneously broken gauge theory. The supersymmetric set of equations of motion (**) for the vector field W^μ, the Dirac spinor E and the real scalar B_t is

$$\begin{cases} \partial_\mu W^{\mu\nu} = (m + eB_t)^2 W^\nu - e \bar{E}_L \gamma^\nu E_L , \\ \Box B_t = - (m + eB_t) \left(mB_t + \frac{eB_t^2}{2} + eW_\mu W^\mu \right) - ie \bar{E} E , \\ i \partial E = - i(m + eB_t) E + e \hat{W} E_L . \end{cases} \tag{32}$$

5. – From supersymmetric to Wess-Zumino gauges.

We have seen in sect. 4 that for $\xi > 0$ the vacuum state is defined by $\langle \exp [2eC] \rangle = 0$, and that supersymmetry is spontaneously broken. At this point the description of the model is singular, and we have to find a way to

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(*) The change of fields (29), (31) reduces to (17) when $e \to 0^+$, $m = ev$ fixed. The origin of the constraint $B > 0$ (which implies no constraint on B_t in this limit) will be understood in sect. 5: it is associated with the use of polar co-ordinates in the (B, A)-plane.

(**) $W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$, $\Box = - \partial_\mu \partial^\mu = \dfrac{\partial^2}{\partial t^2} - \Delta$.

deal with this. Since we cannot only consider the scalar field $B = v \exp[eC]$, we define two scalar fields A and B by

$$A - iB = -iv \exp[e(C + i\Omega)]. \tag{33}$$

Physics will be invariant under a change of function Ω (gauge transformation); at the moment Ω is not fixed. When $\xi > 0$, the vacuum state, defined by $\langle A \rangle = \langle B \rangle = 0$, is not singular for any of these fields; it is gauge invariant.

It is easy to find the transformation, generalizing (29), which makes the Lagrangian density (26) polynomial. Phases are fixed for agreement with other notations:

$$\begin{cases} \eta = -i\dfrac{A - iB}{\sqrt{2}} = -\dfrac{v}{\sqrt{2}} \exp[e(C + i\Omega)], \\ \psi_L = ev\chi_L \exp[e(C + i\Omega)], \\ H = -i\dfrac{F + iG}{\sqrt{2}} = \dfrac{iev}{\sqrt{2}} (M + iN + e\bar{\chi}_R\chi_L) \exp[e(C + i\Omega)], \end{cases} \tag{34a}$$

$$V_\mu = W_\mu - \partial_\mu \Omega. \tag{34b}$$

We recover expression (9) of the Lagrangian density in the Wess-Zumino gauge, where the covariant derivative is defined by

$$iD_\mu = i\partial_\mu - eV_\mu. \tag{35}$$

Transformations (34) leading from the Lagrangian density in the supersymmetric gauge to that in the WZ gauge can be identified with a generalized gauge transformation. To verify that, we perform the following decomposition of the vector multiplet $\mathscr{W}$ (see ref. (4)):

$$\mathscr{W} = \mathscr{V} + \partial\Omega \tag{36}$$

with

$$\mathscr{W} \begin{cases} C, \\ \chi, \\ M, \\ N, \\ W_\mu, \\ \lambda, \\ D, \end{cases} \quad \mathscr{V} \begin{cases} 0, \\ 0, \\ 0, \\ 0, \\ V_\mu = W_\mu - \partial_\mu \Omega, \\ \lambda, \\ D, \end{cases} \quad \partial\Omega \begin{cases} C, \\ \chi, \\ M, \\ N, \\ \partial_\mu \Omega, \\ 0, \\ 0. \end{cases} \tag{37}$$

$\partial\Omega$ is a vector multiplet derived from the scalar multiplet Ω of components (Ω, C, χ, M, N). Let Ω be represented by the left-handed superfield A. $\mathscr{W}$, $\mathscr{V}$ and $\partial\Omega$ are represented respectively by the real superfields W, V and $i(A-A^\dagger)$, related by

$$W = V + i(A - A^\dagger) . \tag{38}$$

Formula (34*a*) can be rewritten

$$S = -\frac{iv}{2}\exp[2ieA] , \tag{39}$$

where the left-handed superfield S represents a scalar multiplet $\mathscr{S}$ of components (φ, ψ_L, H).

Formulae (34), equivalent to the generalized gauge transformation (38), (39), lead from (26), describing the self-interacting vector multiplet $\mathscr{W}$, to formula (9), describing the interaction of the vector multiplet $\mathscr{V}$ with the scalar one $\mathscr{S}$ in the WZ gauge.

6. – Two neutrinos.

6·1. *Goldstone spinor.* – We have seen that, for $\xi > 0$, supersymmetry is spontaneously broken, but ordinary gauge invariance is not; therefore the Goldstone spinor must be the Majorana spinor λ. To verify this explicitly, we perform a supersymmetry transformation of infinitesimal parameter α; we search for $\langle\delta\psi_L\rangle$ and $\langle\delta\lambda\rangle$, vacuum expectation values of the variations of the spinor fields.

If we use the supersymmetric gauge, we know that $(C, \chi, M, N, W_\mu, \lambda, D)$ transform as the components of a vector multiplet

$$\begin{cases} \delta\chi_L = (M + iN + \hat{V} - i\hat{\partial}C)\alpha , \\ \delta\lambda \;\; = (-\tfrac{1}{2}V_{\mu\nu}\gamma^\mu\gamma^\nu + \gamma_5 D)\alpha . \end{cases} \tag{40}$$

Using (34*a*), we find

$$\begin{cases} \langle\delta\psi_L\rangle = \langle ev\exp[e(C + i\Omega)](M + iN)\rangle\alpha = \langle F + iG\rangle\alpha , \\ \langle\delta\lambda\rangle \;\; = \langle D\rangle\gamma_5\alpha . \end{cases} \tag{41}$$

If we choose the WZ gauge, we must restaure it after the infinitesimal supersymmetry transformation, and thus also perform an infinitesimal generalized gauge transformation. The latter can be ignored when computing $\langle\delta\psi_L\rangle$ and $\langle\delta\lambda\rangle$, and we recover the results found in the supersymmetric gauge.

Since

$$\langle D \rangle = -\xi\,, \qquad \langle F \rangle = \langle G \rangle = 0\,, \tag{42}$$

the Goldstone spinor is λ, as expected.

6·2. *Two conserved quantum numbers.* – λ is massless from Goldstone theorem and ψ_L from gauge invariance. Both results can be obtained from invariance (*) of the broken theory under the transformation $L(\alpha, \beta)$:

$$\begin{cases} LV(x, \theta, \bar{\theta}) L^{-1} = V(x, \theta \exp[-i\alpha], \bar{\theta} \exp[i\alpha])\,, \\ LS(x, \theta, \bar{\theta}) L^{-1} = \exp[i(\alpha + \beta)]\, S(x, \theta \exp[-i\alpha], \bar{\theta} \exp[i\alpha])\,. \end{cases} \tag{43}$$

This gives for components

$$\begin{cases} V_\mu \to V_\mu\,, \\ \varphi \to \exp[i(\alpha + \beta)]\varphi\,, \end{cases} \qquad \begin{cases} \lambda_L \to \exp[i\alpha]\,\lambda_L\,, \\ \psi_L \to \exp[i\beta]\,\psi_L\,. \end{cases} \tag{44}$$

We can define two « leptonic » numbers conserved separately. The « electronic » and « muonic » numbers are, respectively,

$$V_\mu(0, 0)\,, \quad \lambda_L(1, 0)\,, \quad \psi_L(0, 1)\,, \quad \varphi(1, 1)\,. \tag{45}$$

6·3. *Two neutrinos.* – If we want to interpret λ_L and ψ_L as neutrino fields, we must take the limit $e \to 0^+$. The limit $e \to 0^+$, $\xi \to -\infty$ (ξe fixed) gave us the free massive vector multiplet: supersymmetry conserved, gauge invariance explicitly broken. Similarly, the limit $e \to 0^+$, $\xi \to +\infty$ (ξe fixed) gives a free model, still gauge invariant, but breaking explicitly supersymmetry.

The particles described are interpreted as the photon γ, the two neutrinos ν_e and ν_μ and a complex scalar ω. This model is free, but we can consider it as a building block to be incorporated in realistic ones.

Since γ, ν_e and ν_μ are massless, it would not be very surprising if the scalar ω remained massless, or acquired only a small mass. We have to verify that ω does not modify the known phenomenology of weak and electromagnetic interactions. ω is neutral, and has electronic number 1, muonic number 1, ω exchanges conserve the difference between leptonic numbers, and cannot induce processes such as μ-decay, or $K \to \mu^\pm e^\mp$, $\pi \to \mu^\pm e^\mp$.

(*) $L(\alpha, \beta)$ acts on the superfields V and S as the Q and R transformations used in sect. **5** of ref. ([3]).

We constructed in ref. ([3]) a supersymmetric model of weak and electromagnetic interactions, but for the electron sector only. ν_e was the Goldstone spinor, associated with γ in a massless vector multiplet. The model introduced a heavy electron E_-, of leptonic number -1, coupled to the electron through a complex scalar ω of leptonic number 2.

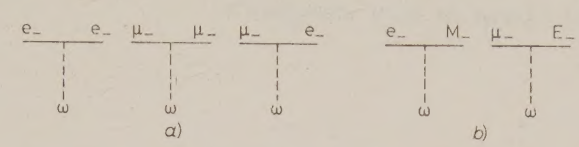

Fig. 1. – *a*) Forbidden couplings of ω with known charged leptons. *b*) Possible couplings of ω with charged leptons.

In a theory including the muon sector, ω would have electronic number 1 and muonic number 1; couplings with the electron and heavy muon, or with the muon and heavy electron, may appear (see fig. 1). The existence of such a (possibly light) ω does not seem to cause any trouble with known facts.

7. – Conclusion.

This model describes the supersymmetric gauge-invariant interaction of a vector multiplet with a left-handed scalar one. Two different gauges have been used, and their relations studied in detail: the Wess-Zumino gauge and a supersymmetric gauge, in which only the vector multiplet $\mathscr{W}$ survives; then the Lagrangian density reads

$$\mathscr{L} = \mathscr{L}_0 + \left[\xi \mathscr{W} + \frac{v^2}{4} \exp\left[2e\mathscr{W}\right]\right]_D ,$$

where the parameter v has no physical meaning.

Among all possible (mass + self-interaction) terms for a vector multiplet of the form $[f(\mathscr{W})]_D$, we recovered the previous one, simply requiring that terms quartic with respect to the spinor fields eliminate.

There are two situations of spontaneous symmetry breaking. $\xi < 0$ corresponds to the supersymmetric extension of the Higgs model, describing the interaction of a vector, a Dirac spinor and a real scalar. It depends only on the common mass m and the coupling constant e: supersymmetry allows one to reduce the number of arbitrary parameters in a spontaneously broken gauge theory.

For $\xi > 0$ supersymmetry is spontaneously broken, but gauge invariance is not; the model describes a vector, two left-handed Dirac spinors, all mass-

less, and a complex scalar field, massive. If $e \to 0^+$, the model becomes free: the fields can be used to represent the photon, the two neutrinos and a scalar particle ω. It remains to embed $(\gamma, \nu_e, \nu_\mu, \omega)$ in a realistic theory of weak and electromagnetic interactions.

ω (which may be light) is a weak-interacting scalar particle with electronic number 1 and muonic number 1; its possible existence does not lead to important effects except at high energies (*).

APPENDIX

It may be useful to recall the invariances of the model presented in sect. **5** of ref. ([3]), in the absence of the soft parity-breaking term $\xi D'$ (see subsect. **5**·4). $SU_2 \times U_1$ is the gauge group, but others can be considered easily. Besides supersymmetry, parity and «R-invariance» are required. Those are conserved by spontaneous breaking, while gauge invariance is not.

The model preserves both supersymmetry and parity, and a conserved quantum number (sometimes called «fermionic number») is associated with R-invariance. This is valid for the non-Abelian as well as the Abelian case; for example, if SU_2 is the gauge group, the model describes the supersymmetric, parity and R-invariant interaction of three vector multiplets, of mass $gv/\sqrt{2}$, with a complex scalar multiplet of mass $hv/\sqrt{2}$; if U_1 is the gauge group, we have only one massive vector multiplet, interacting with a complex scalar one.

(*) When completing this paper we learned that the interaction between a vector and a left-handed scalar superfield has also been considered by G. B. MAINLAND and K. TANAKA (Ohio State University).

RIASSUNTO (*)

Il modello di Higgs può essere reso supersimmetrico purché si aggiunga uno spinore di Dirac, ed il termine quartico d'interazione nella densità lagrangiana sia $-(e^2/2)(\varphi^\dagger \varphi)^2$, in cui e è la costante di accoppiamento di gauge. Il modello descrive un vettore, uno spinore di Dirac ed uno scalare reale; esso dipende solo da due parametri arbitrari, m ed e. Come modello invariante di gauge esso può essere studiato in una gauge di Wess-Zumino od in una gauge supersimmetrica. Esso può anche essere ottenuto come una teoria di un multipletto vettoriale autointeragente. Infine si suggerisce che una particella scalare può essere associata al fotone ed ai due neutrini nella teoria delle interazioni deboli ed elettromagnetiche.

(*) *Traduzione a cura della Redazione.*